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EXISTENCE OF A SOLUTION TO A MIXED PROBLEM FOR A PARABOLIC EQUATION IN THE SENSE OF SHILOV

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ABSTRACT

Mixed problem for the fourth order ordinary differential equation with general boundary conditions is considered in present paper. Soluion of the problem is found by the residue method. According to the scheame of this method the mixed problem is divided by two auxiliary- specrtal and Cauchy problems. After researching these two problems, solution of the considering mixed problem is found by residue series. It is shown , that solution of considering mixed problem surround not only parabolic equations in the sense of Shilov, but also wider classes of equations.

Introduction

The fourth-order harmonic Schrödinger equation is of great importance in the study of wave processes. These equations can be transformed into parabolic type equations in the sense of Petrovsky [4,5,11,12]. Additionally, there are more general forms of parabolic equations beyond those defined by Petrovsky, such as parabolic equations in the sense of Shilov [7,8,9,10].

İn equations of this type, the inclusion of the potential function can alter the nature of the equation. In other words, wave processes can be analyzed within the framework of parabolic equations in the sense of Shilov, and the fourth-order equation we consider falls into this class. [1]

For instance, examining the heat transfer process in rods of the same length but with different heat transfer coefficients can be represented by fourth-order partial differential equations [5,6].

The fourth-order parabolic equations in the sense of Shilov.

Consider the following problem:

$$\frac{\partial u(x,t)}{\partial t} = ip\frac{\partial^4 u(x,t)}{\partial x^4} + e^{ix}\frac{\partial^2 u(x,t)}{\partial x^2}, \ 0 < x < 1, \ t > 0 \tag{1}$$

$$u(x,0) = \varphi(x), \tag{2}$$

$$L_m(u) \equiv \sum_{k=1}^{4}\left(\alpha_{mk} \frac{\partial^{k-1}u(0,t)}{\partial^{k-1} x} + \beta_{mk} \frac{\partial^{k-1}u(1,t)}{\partial^{k-1}x} \right) = 0, \quad m=\overline{1,4} \tag{3}$$

where α_{mk}, β_{mk} ($m=\overline{1,4}$, $k=\overline{1,4}$) are complex numbers, $p>0$ *real number*, $\varphi(x)$ is complex valued functions.

After application integral transformation

$$y(x,\lambda) = \int_0^\infty u(x,t)e^{-\lambda^4 t}dt$$

to the problem (1)-(3), we'll get following spectral problem:

$$ipy^{IV} + q(x)y'' - \lambda^4 y = -\phi(x), \quad 0<x<1 \tag{4}$$

$$L_m(y) \equiv \sum_{k=1}^{4}\left(\alpha_{mk} y^{(k-1)}(0,\lambda) + \beta_{mk} y^{(k-1)}(1,\lambda)\right) = 0, \quad m=\overline{1,4}\ . \tag{5}$$

Roots of the characteristic equation in the sense of Birkhof corresponding to the equation (4) are found as follows [2]:

$$\theta_1 = p^{-\frac{1}{4}} \cdot e^{-\frac{\pi}{8}i}, \ \theta_2 = i\,\theta, \ \theta_3 = -\theta_1, \ \theta_4 = -i\,\theta_1\ .$$

To find asymptotic of fundamental solutions of the equation (4) let's devide a complex plane λ into eight sectors by the following way [7]:

$$S_k = \left\{ \lambda : -\lambda_1\, tg\frac{\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{\pi}{8} \right\}, \ k=1,2,$$

$$S_k = \left\{ \lambda : \lambda_1\, tg\frac{\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{3\pi}{8} \right\}, \ k=3,4,$$

$$S_k = \left\{ \lambda : \lambda_1\, tg\frac{3\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{5\pi}{8} \right\}, \ k=5,6,$$

$$S_k = \left\{ \lambda : \lambda_1\, tg\frac{5\pi}{8} < (-1)^{k-1}\lambda_2 < \lambda_1 tg\frac{7\pi}{8} \right\}, \ k=7,8.$$

At the each sectors $S_k\ (k=\overline{1,8})$ at large values of $|\lambda|$ the asymptotics of fundamental solution of the equation (4) have the following representation [6, 8]:

$$\frac{d^s y_n(x,\lambda)}{dx^s} = (\lambda\theta_n)^s \left[1 + \frac{1}{4i\theta_n\lambda}(e^{ix}-1) + O\left(\frac{1}{\lambda^2}\right) \right] e^{\lambda\theta_n x}, \tag{6}$$
$$|\lambda| \to +\infty, \ \lambda \in S_m\ (m=\overline{1,8}), \ n=\overline{1,4}, \ s=\overline{0,3}.$$

Green function of the spectral problem (4), (5) has the form [4] :

$$G(x,\xi,\lambda) = \frac{\Delta(x,\xi,\lambda)}{\Delta(\lambda)}\ ; \ \lambda \in S_m, \ m=\overline{1,8}. \tag{7}$$

$\Delta(\lambda)$ is called a characteristic determinant and is found as follows

$$\Delta(\lambda)=\begin{vmatrix} L_1(y_1) & L_1(y_2) & L_1(y_3) & L_1(y_4) \\ L_2(y_1) & L_2(y_2) & L_2(y_3) & L_2(y_4) \\ L_3(y_1) & L_3(y_2) & L_3(y_3) & L_3(y_4) \\ L_4(y_1) & L_4(y_2) & L_4(y_3) & L_4(y_4) \end{vmatrix}$$

(8) and auxiliary determinant $\Delta(x,\xi,\lambda)$ is found as follows

$$\Delta(x,\xi,\lambda)=\begin{vmatrix} g(x,\xi,\lambda) & y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & y_4(x,\lambda) \\ L_1(g)_x & L_1(y_1) & L_1(y_2) & L_1(y_3) & L_1(y_4) \\ L_2(g)_x & L_2(y_1) & L_2(y_2) & L_2(y_3) & L_2(y_4) \\ L_3(g)_x & L_3(y_1) & L_3(y_2) & L_3(y_3) & L_3(y_4) \\ L_4(g)_x & L_4(y_1) & L_4(y_2) & L_4(y_3) & L_4(y_4) \end{vmatrix},$$

(9)

where Cauchy function $g(x,\xi,\lambda)$ is found as follows [5]

$$g(x,\xi,\lambda)=\pm\frac{1}{2}\sum_{k=1}^{4} z_k(\xi,\lambda)y_k(x,\lambda)$$

"+" if $0\le\xi\le x\le 1$, "-" if $0\le x\le\xi\le 1$,

here $z_k(\xi,\lambda)=\dfrac{V_{4k}(\xi,\lambda)}{V(\xi,\lambda)}$, $k=\overline{1,4}$,

$V_{4k}(\xi,\lambda)$ is an algebraic complement of the fourth row element of Vronskian $V(\xi,\lambda)$.

To find the asymptotic of eigenvalues of spectral problem (4), (5) let's introduce the following notations :

$$L(\gamma_{k_1}^1\gamma_{k_2}^2\gamma_{k_3}^3\gamma_{k_4}^4)=\begin{vmatrix} \gamma_{1k_1}^1 & \gamma_{1k_2}^2 & \gamma_{1k_3}^3 & \gamma_{1k_4}^4 \\ \gamma_{2k_1}^1 & \gamma_{2k_2}^2 & \gamma_{2k_3}^3 & \gamma_{2k_4}^4 \\ \gamma_{3k_1}^1 & \gamma_{3k_2}^2 & \gamma_{3k_3}^3 & \gamma_{3k_4}^4 \\ \gamma_{4k_1}^1 & \gamma_{4k_2}^2 & \gamma_{4k_3}^3 & \gamma_{4k_4}^4 \end{vmatrix},$$

$A_0=2L(\alpha_2\alpha_3\beta_2\beta_3)$,

$B_0=2\big(L(\alpha_2\alpha_3\beta_1\beta_3)-L(\alpha_1\alpha_3\beta_2\beta_3)\big)$,

$C_0=2\big(L(\alpha_1\alpha_2\alpha_3\beta_3)+L(\alpha_3\beta_1\beta_2\beta_3)\big)$,

$g_k(x)=\dfrac{1}{4\theta_k}(e^{ix}-1), k=\overline{1,4}.$

Now to find asymptotic of eigenvalues of spectral problem (4), (5) consider the following theorem:

Theorem1. Suppose,that α_{mk}, β_{mk} $(m=\overline{1,4}\ ;\ k=\overline{1,3})$ are complex numbers. Then the zeros of the characteristic determinant $\Delta(\lambda)$ are countable set, single limit points of which is $\lambda=\infty$ and the following formulas for the asymptotic zeros are true:

$$\lambda_n^4=\left(4n^4+8n^3+6n^2\right)\pi^4 i-\pi^2 n^2\left(\sin 1-i(\cos 1-1)+4i\frac{B_0\pm C_0}{A_0}\right)+O(n)\quad n\to\pm\infty \tag{10}$$

Proof

Based on the property of determiant, the $\Delta(\lambda)$, found by formula (8) can be rewritten as follows:

$$\Delta(\lambda)=D_{12}(\lambda)e^{(\theta_1+\theta_2)\lambda}+D_{14}(\lambda)e^{(\theta_1+\theta_4)\lambda}+D_{23}(\lambda)e^{(\theta_2+\theta_3)\lambda}+D_{34}(\lambda)e^{(\theta_3+\theta_4)\lambda}+ \\ \cdots\cdots\cdots+\sum_{k=1}^{4}D_k(\lambda)e^{\theta_k(\lambda)}+D_0(\lambda). \tag{11}$$

To find the main part of determinant $\Delta(\lambda)$ let's use the traditional method, that is equate the real part of exponents in pairs and selecting the straight lines or semi-straights, we'll get [1,3]:

$$\lambda_2=\lambda_1 tg\left(\frac{\pi}{8}+\frac{\pi}{4}(k-1)\right),\ k=\overline{1,8},\quad |\lambda_1|>R.$$

Choose those of the semi-strips, constructed from the above-mentioned semi-straights, where the main part of $\Delta(\lambda)$ has an infinite number of zeroes. Let's denote as $\Pi_k(\lambda)$ and $\Delta_k(\lambda)$ $(k=\overline{1,4})$ these kinds of semi-strips and defined there the main parts of the determinant $\Delta(\lambda)$, correspondingly. Thus, the semi-strips $\Pi_k(\lambda)$ and $\Delta_k(\lambda)$ -the main part of $\Delta(\lambda)$ are defined as follows:

Consider the main part of $\Delta(\lambda)$ in the first quarter [4]:

$$\Pi_1(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{\pi}{8}<\delta,\ \lambda_1>R\right\},$$

$$\Delta_1(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1 A_0\left(1+(1-i)g_1(1)\frac{1}{\lambda}\right)+(1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_2)\lambda}+\right. \\ \left.+\left(i\theta_1 A_0\left(1+(1+i)g_1(1)\frac{1}{\lambda}\right)+(-1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_4)\lambda}+\left(2iC_0+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_1\lambda}\right].$$

In the second quarter the main part has the form:

$$\Pi_2(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{5\pi}{8}<\delta,\ \lambda_1>R\right\}$$

$$\Delta_2(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1 A_0\left(1+(1-i)g_1(1)\frac{1}{\lambda}\right)+(1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_2)\lambda}+\right. \\ \left.+\left(i\theta_1 A_0\left(1+(-1-i)g_1(1)\frac{1}{\lambda}\right)+(1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_2+\theta_3)\lambda}+\left(2iC_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_2\lambda}\right].$$

The main part in the third quarter has the form

$$\Pi_3(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{\pi}{8}<\delta,\ \ \lambda_1<-R\right\}$$

$$\Delta_3(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1A_0\left(1+(-1-i)g_1(1)\frac{1}{\lambda}\right)+(1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_2+\theta_3)\lambda}+\right.$$

$$\left.+\left(i\theta_1A_0\left(1+(-1+i)g_1(1)\frac{1}{\lambda}\right)+(-1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_3+\theta_4)\lambda}+\left(-2iC_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_3\lambda}\right]$$

$$\Pi_4(\lambda)=\left\{\lambda=\lambda_1+i\lambda_2:-\delta<\lambda_2-\lambda_1 tg\frac{5\pi}{8}<\delta,\ \ \lambda_1<-R\right\}$$

$$\Delta_4(\lambda)=\theta_1^9\lambda^{10}\left[\left(i\theta_1A_0\left(1+(1+i)g_1(1)\frac{1}{\lambda}\right)+(-1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_1+\theta_4)\lambda}+\right.$$

$$\left.+\left(i\theta_1A_0\left(1+(-1+i)g_1(1)\frac{1}{\lambda}\right)+(-1-i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{(\theta_3+\theta_4)\lambda}+\left(-2C_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda^2}\right)\right)e^{\theta_4\lambda}\right],$$

here $\delta>0$ and R is sufficiently large number.

Firstly, solve equation $\Delta_1(\lambda)=0$. For that introduce following notations:

$$\Delta_{11}(\lambda)=\left(i\theta_1A_0e^{\theta_2\lambda}+i\theta_1A_0e^{\theta_4\lambda}\right)\theta_1^9\lambda^{10}e^{\theta_1\lambda},$$

$$\Delta_{10}(\lambda)=\Delta_1(\lambda)-\Delta_{11}(\lambda)=\theta_1^9\lambda^{10}\left(i\theta_1A_0(1-i)g_1(1)+(1+i)B_0\frac{1}{\lambda}+O\left(\frac{1}{\lambda}\right)\right)e^{(\theta_1+\theta_2)\lambda}+$$

$$+\left(i\theta_1A_0(1+i)g_1(1)+(-1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{(\theta_1+\theta_4)\lambda}+\left(2iC_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_1\lambda}\Bigg].$$

$\mu_{1n}=\left(\frac{\pi}{2}+\pi n\right)\frac{1}{\theta_1}$, $n\to+\infty$. After solution of the equation $\Delta_{11}(\lambda)=0$ we will get:

To find roots of the equation $\Delta_1(\lambda)=0$ consider a following formulas [9]

$$\lambda_{1n}^r=\mu_{1n}^r+r\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\underset{\lambda=\mu_{1n}}{res}\left|\lambda^{r-1}\left(\frac{\Delta_{10}(\lambda)}{\Delta_{11}(\lambda)}\right)^m\right|.$$

In case of $r=4$ and $m=1$ we will get

$$\lambda_{1n}^4=\mu_{1n}^4-4\underset{\lambda=\mu_{1n}}{res}\lambda^3\frac{\Delta_{10}(\lambda)}{\Delta_{11}(\lambda)}=\mu_{1n}^4-4\underset{\lambda=\mu_{1n}}{res}\lambda^2\times\left[\left(i\theta_1A_0(1-i)g_1(1)+(1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_2\lambda}+\right.$$

$$\left.+\left(i\theta_1A_0(1+i)g_1(1)+(-1+i)B_0+O\left(\frac{1}{\lambda}\right)\right)e^{\theta_4\lambda}+\left(2iC_0+O\left(\frac{1}{\lambda}\right)\right)\right]/\left(i\theta_1A_0e^{\theta_2\lambda}+i\theta_1A_0e^{\theta_4\lambda}\right).$$

It is easy to check, that μ_{1n} are simple poles of the function $\Delta_{11}(\lambda)$. According to that, we'll get

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2 \frac{\theta_1 A_0 g_1(1)\left((1-i)e^{\theta_2\mu_{1n}} + (-1+i)e^{\theta_4\mu_{1n}}\right)e^{\theta_2\lambda} + B_0\left((1+i)e^{\theta_2\mu_{1n}} + (-1+i)e^{\theta_4\mu_{1n}}\right) + 2iC_0}{i\theta_1\theta_2 A_0\left(e^{\theta_2\mu_{1n}} - e^{\theta_4\mu_{1n}}\right)},$$

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2\left[\frac{g_1(1)}{i\theta_2} + \frac{B_0}{i\theta_1\theta_2 A_0} - \frac{2C_0}{\theta_1\theta_2 A_0\left(e^{\theta_2\mu_{1n}} - e^{-\theta_4\mu_{1n}}\right)}\right],$$

$$\lambda_{1n}^4 = \mu_{1n}^4 - 4\mu_{1n}^2\left[\frac{g_1(1)}{i\theta_2} + \frac{B_0}{i\theta_1\theta_2 A_0} - \frac{C_0}{\theta_1\theta_2 A_0 \sin\left(\frac{\pi}{2} + \pi n\right)}\right],$$

$$\lambda_{1n}^4 = \mu_{1n}^4 + 4\mu_{1n}^2\left[\frac{g_1(1)}{\theta_1} + \frac{B_0}{\theta_1^2 A_0} + \frac{C_0}{\theta_1^2 A_0}(-1)^{(n-1)}\right].$$

Taking into account μ_{1n} and $g_1(1)$ into the last equality, we'll get:

$$\lambda_{1n}^4 = \left(4n^4 + 8n^3 + 6n^2\right)\pi^4 i - \pi^2 n^2\left(\sin 1 - i(\cos 1 - 1) + 4i\frac{B_0 + (-1)^{n-1}C_0}{A_0}\right) + O(n),\ n \to +\infty.$$

After solution the equation $\Delta_k(\lambda) = 0$ $(k = 2,3,4)$ by the same way we'll get following asymptotic formulas:

$$\lambda_{2n}^4 = \mu_{2n}^4 - 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^{n-1}C_0}{\theta_1^2 A_0}\right]\mu_{2n}^2,$$

$$\mu_{2n} = \frac{(1+2n)\pi i}{2\theta_1},\ n \to -\infty,$$

$$\lambda_{3n}^4 = \mu_{3n}^4 + 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^n C_0}{\theta_1^2 A_0}\right]\mu_{3n}^2,$$

$$\mu_{3n} = \frac{(1+2n)\pi}{2\theta_1},\ n \to -\infty,$$

$$\lambda_{4n}^4 = \mu_{4n}^4 - 4\left[\frac{g_1(1)}{\theta_1} + \frac{B_0 + (-1)^n C_0}{\theta_1^2 A_0}\right]\mu_{4n}^2,$$

$$\mu_{4n} = \frac{(1+2n)\pi i}{2\theta_1},\ n \to +\infty.$$

Substituting μ_{kn} $(k = 2,3,4)$ into equalities for λ_{kn}^4 $(k = 2,3,4)$ we'll get formula (10). The theorem is proved.

As it is known, that at equation (1) is parabolic in the sense of Shilov [10]. A following theorem allows us to find solution of the mixed problem (1)-(3) not only in case of parabolic in the sense of Shilov, but also wider classes:

Theorem 2. Suppose ,that function $\varphi(x)$ are satisfies to a following conditions $\varphi(x)\in C^2[0,1]$, $\varphi(0)=\varphi(1)=\varphi'(0)=\varphi'(1)=0$ .If $A_0\neq 0$, coefficients of the boundary conditions are complex numbers and $\mathrm{Im}\frac{B_0\pm C_0}{A_0}\leq\frac{1}{4}\sin 1$, then mixed problem (1)-(3) has following solution

$$u(x,t)=-i\sum_{k=1}^{4}\sum_{n=1}^{\infty}\underset{\lambda=\lambda_{km}}{res}\ \lambda^3 e^{\lambda^4 t}\int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi\,, \tag{12}$$

here $G(x,\xi,\lambda)$ is a Green function of the corresponding spectral problem , λ_{kn} $(k=\overline{1,4}; n=1,2,3,...)$ are all zeroes of the characteristic determinant $\Delta(\lambda)$.

Proof. Let's search solution of the mixed problem (1)-(3) as follows

$$u(x,t)=\sum_{k=1}^{4}\sum_{n=1}^{\infty}\underset{\lambda=\lambda_{km}}{res}\ \lambda^3\int_0^1 G(x,\xi,\lambda)z(\xi,t,\lambda)d\xi\,. \tag{13}$$

Taking into account (12) into (1) and (2) we can find function $z(\xi,t,\lambda)$ in such form

$$z(\xi,t,\lambda)=-ie^{\lambda^4 t}\varphi(\xi)..$$

Taking into account the last into (13), we will get formula (12).

From condition $A_0\neq 0$ can be said, that problem (4), (5) is regular [4,5]. It means, that out of $\delta>0$ neighborhood of zeros of the characteristic determinant $\Delta(\lambda)$ inequality

$$\left|G^{(k)}(x,\xi,\lambda)\right|\leq\frac{M_k(x,\xi,\lambda)}{|\lambda|^{3+k}},\ k=\overline{0,3},\ \lambda\in S_j\ (j=\overline{1,8}) \tag{14}$$

is true, where $M_k(x,\xi,\lambda)$ are positive, bounded with respect to x and ξ functions and analytic function with respect to λ-complex parameter. At the same time, under condition $A_0\neq 0$ and $\varphi(x)\in C^2[0,1]$, $\varphi(0)=\varphi(1)=\varphi'(0)=\varphi'(1)=0$ for the function $\varphi(x)$ following formula of decomposition is true [5]:

$$\varphi(x)=-i\sum_{k=1}^{4}\sum_{n=1}^{\infty}\underset{\lambda=\lambda_{kn}}{res}\ \lambda^3\int_0^1 G(x,\xi,\lambda)\varphi(\xi)d\xi\,. \tag{15}$$

Taking into account (15) we can see that series given by formula (12) satisfies to initial condition. As the Green function $G(x,\xi,\lambda)$ is a solution of the homogeneous equation, corresponding to (4), (5), the series (12) formally satisfies to the boundary condition (2).

It is necessary for (12) and series, obtaining by differentiating it four times with respect to x, and ones with respect to convergence uniformly and absolutely. For that, taking into account conditions of the theorem and asymptotic of eigenvalues, defined by formula (10) we'll get:

$$\left|e^{t\lambda_{kn}^4}\right|=e^{t\,\mathrm{Re}\,\lambda_{kn}^4}=e^{-t\pi^2 n^2\left(\sin 1-4\,\mathrm{Im}\frac{B_0\pm C_0}{A_0}\right)+O(n)}\,.$$

It shows, that if $A_0 \neq 0, \operatorname{Im}\frac{B_0 \pm C_0}{A_0} \leq \frac{1}{4}\sin 1$ formula satisfies according to Weierstrass theorem the functional series (12) uniformly and absolutely convergence. It means that our search formal operations are justified.

The theorem is proved.

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