

# CYLINDRICAL SHAPE OF FREE OSCILLATION OF A RECTANGULAR ORTHOTROPIC PLATE LOCATED ON AN INHOMOGENEOUS VISCOELASTIC BASE

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Abstract- The presented paper is devoted to the solution of cylindrical free oscillation problems of a rectangular plate, taking into account the resistance of the external environment. In the title of the article, accepting a cylindrical oscillation is not chosen randomly and it is a rare issue in the theory of plates and coatings, therefore, the solution of the considered problem is new and relevant. In this paper, the influence of the external environment is considered viscoelastic according to the Voigt model, the plate is assumed to be rectangular, thin-walled, and orthotropic, being heterogeneous along its length (i.e., in the direction of the large side). The conditions for fixing the contours of the plate are chosen in such a way that a heterogeneous boundary condition is obtained, otherwise the shape of the free oscillation cannot be cylindrical. Under the required conditions, the oscillation equation of the plate is formulated, and as a result, the fourth order partial differential equation with a variable coefficient is obtained. The obtained equation is a complicated enough equation modified from the Sophie-Germain equation written for the deflection of the plates according to the condition of the problem, and it is hard to be solved. Such equations do not have an analytical solution, and for now the most rational method for its solution is considered to be Bunov Galerkin's method of orthogonalization and separation of variables. In order to calculate the value of the frequency of the oscillation, the relationship equations were obtained, the calculations were carried out for cases where the characteristic functions change with a linear law, and the material of the plate is homogeneous and heterogeneous. Equations of dependence between dimensionless frequency and characteristic functions and parameters characterizing a heterogeneous viscoelastic base were obtained. An error function was constructed for the obtained equation and the orthogonalization condition was checked with the help of the error function. In order to compare the solution of the problem, the frequency analysis of the plate was carried out using the finite element method, for which the frequency analysis was

carried out in 6 frequency cases on plates with different length dimensions. Solidworks software was used here. The results were presented in form of graphs and tables.

**Keywords:** Rectangular Plate, Free Cylindrical Oscillation, Viscoelastic Base, Heterogeneous Plate, Frequency.

## **1. INTRODUCTION**

Rectangular plates made of various materials are widely used in modern engineering devices, machine building, shipbuilding and many other fields. These boards are mainly made of natural and artificial materials. Depending on the production conditions, the mechanical properties and density of the boards are variable along the thickness and the length. In the calculations, it is mainly assumed that the plates are made of a homogeneous material. During operation, boards are also exposed to various types of the external environment. Taking into account the influence of the external environment in solving engineering problems was first offered by the German engineer Emil Winkler in the 19th century. This offered model consists of a hydrostatic influence. In the following years, new models were offered that kept the Winkler model within them and were considered more accurate. A model that fully expresses the physical and mechanical (viscoelastic and heterogeneity) properties of the environment is used in the offered paper. A brief summary of the works related to the considered problem is presented in the paper.

In [1], free oscillation motion problems of rectangular plates located on a viscoelastic base and a base characterized by the Winkler model were studied. Here, it is accepted the assumption that the material has different resistance to extension and compression. Relationship equations between the dimensionless frequency and parameters characterizing the resistance of the external environment were obtained. In papers [2, 3], oscillation problems of plates located on the base characterized by the Winkler model, and homogeneous along the thickness and length, were solved. In addition, oscillation problems of a plate with a circular cross-section and located on a heterogeneity base were studied. In paper [4], the transverse oscillations of a beam made of multinodular material and located in a heterogeneous viscoelastic environment were studied. In papers [5, 6], viscoelastic free and geometric nonlinear oscillations of reinforced cylindrical and conical covers were investigated. In papers [7, 9], free oscillations of rectangular plates used in construction of oil and gas facilities, flat stability of an elastic, plastic beam, and stability in the case of different resistance to extension and compression were considered. In [10], the optimal parameters of heterogeneous structural elements were investigated by means of experimental research models and technologies.

In [11], free oscillations of a vertical column consisting of three orthotropic, cylindrical panels in contact with viscoelastic soil and reinforced with longitudinal ribs were investigated. [12] investigated free oscillations of light retaining walls consisting of viscoelastic soil in contact with orthotropic cylindrical shells. In [13], free oscillations of retaining walls consisting of reinforced orthotropic cylindrical shells in contact with the soil were investigated. In [14], the problems of free oscillations of a fluid-filled heterogeneous spherical shell reinforced with heterogeneous toughness ribs in soil were solved. In [15], the issue of substantiating the initial propositions of the methodology for modeling the corrosion degradation of construction elements (plates and pivots) in an aggressive environment was considered.

It can be seen from the analyzed literature that one of the most important problems faced by engineers and designers is to correctly estimate the mechanical properties of the material of plates, the frequency-amplitude characteristics and to take into account the resistance of the external environment in the calculations. In the presented paper, the problem of a free oscillation motion of a rectangular plate located on a heterogeneous base, whose modulus of elasticity and density depend on the spatial coordinate, was studied.

#### 2. PROBLEM STATEMENT

Suppose that the elasticity moduli and densities of the plate and the basis change along the length. For this case, the following dependencies can be written:

$$E_{1}^{1} = E_{1}^{0} f_{1}(x); \quad E_{2}^{2} = E_{2}^{0} f_{1}(x)$$

$$G = G^{0} f_{1}(x); \quad \rho_{1} = \rho_{0} \Psi_{1}(x)$$

$$v_{1}^{1} = \text{const}; \quad v_{2}^{2} = \text{const}$$
(1)

where,  $E_1^0$ ,  $E_2^0$ ,  $G^0$ ,  $\rho_0$  are elasticity modulus, elasticity modulus at sliding and density for homogeneous materials,  $f_1(x)$  is function shows heterogeneity of the rectangular plate along the length, and  $\Psi_1(x)$  is a continuous function characterizing the change in density. The mathematical model for a heterogeneous viscoelastic basis can be written as follows:

$$R = R_1(x) + R_2(x)\frac{\partial^2 W}{\partial t^2}$$
<sup>(2)</sup>

Taking into account the resistance of the external environment, the equation of oscillation motion of a rectangular plate in a cylindrical form can be written as:

$$f_{1}(x)\frac{\partial^{4}W_{1}}{\partial x^{4}} + 2f'(x)\frac{\partial^{3}W_{1}}{\partial x^{3}} + f''(x)\frac{\partial^{2}W_{1}}{\partial x^{2}} + \overline{R_{1}(x)}W_{1} + \overline{R_{2}(x)}\frac{\partial^{2}W_{1}}{\partial t^{2}} + \overline{\rho}\Psi_{1}(x)\frac{\partial^{2}W_{1}}{\partial t^{2}} = 0$$
(3)

The following substitutions are accepted here:

$$\overline{R_{1}(x)} = R_{1}(x) (K_{1}^{0})^{-1}, \ \overline{R_{2}(x)} = R_{2}(x) (K_{1}^{0})^{-1}$$

$$\overline{\rho} = \rho_{1} (K_{1}^{0})^{-1}$$
(4)

where,  $K_1^0$  shows the flexion hardness of a homogeneous orthotropic rectangular plate in the direction of the coordinate axis x. It is calculated by the following expression:

$$K_1^0 = \frac{E_1^0 b^3}{12 \left(1 - v_1^0 v_2^0\right)} \tag{5}$$

where, *b* is the thickness of the rectangular orthotropic plate, and  $v_1^0$ ,  $v_2^0$  are Poisson's coefficients. These are selected from the table for concrete material types.

Since Equation (3) is a complicated partial differential equation of the fourth order, it is difficult to find its exact mathematical solution. Therefore, we solve the problem using orthogonalization method and by the method of expansion in variables (it is assumed that the boundary conditions are homogeneous).

#### **3. PROBLEM SOLUTION**

At the first stage, we look for the solution of Equation (3) as follows:

$$W_1(x,t) = V_1(x)e^{i\omega_1 t} \tag{6}$$

where, the function  $V_1(x)$  is must satisfy the homogeneous boundary conditions for the rectangular plate, and  $\omega_1$  is represents the frequency. If we substitute the expression (6) of the flexion in the Equation (3), we get the following the fourth order differential equation with variable coefficients:

$$f_{1}(x)\frac{d^{4}V_{1}}{dx^{4}} + 2f'(x)\frac{d^{3}V_{1}}{dx^{3}} + f''(x)\frac{d^{2}V_{1}}{dx^{2}} + \overline{R_{1}}(x)V_{1} - -\omega_{1}^{2}\left(\overline{R_{2}(x)} + \overline{\rho}\Psi_{1}(x)\right)V_{1}(x) = 0$$
(7)

As it can be seen, the Equation (7) is also complicated and we find its solution using Bubnov Galerkin's orthogonalization method. We look for the characteristic function  $V_1(x)$  as follows:

$$V_{1}(x) = \sum_{i=1}^{n} b_{i}^{0} \varphi_{i}(x)$$
(8)

where,  $b_i^0$  are constants with unknown values, each of  $\varphi_i(x)$  is must satisfy the homogeneous boundary conditions for a rectangular plate.

Using Equations (7) and (8), the error function can be written as follows:

$$\gamma(x) = \sum_{i=1}^{n} b_i^0 \left[ f_1(x) \frac{d^4 \varphi_i}{dx^4} + 2f'(x) \frac{d^3 \varphi_i}{dx^3} + f''(x) \frac{d^2 \varphi_i}{dx^2} + \overline{R_1(x)} \varphi_i(x) - \omega_1^2 \left[ \overline{R_2(x)} + \overline{\rho} \Psi_1(x) \right] \varphi_i \neq 0$$
(9)

Orthogonality satisfies the following condition:

$$\int_{0}^{a} \gamma(x) \varphi_{q}(x) dx = 0 \quad q = \overline{1, n}$$
(10)

In any approximation  $\omega_1^2$  is found from a system of linear homogeneous equations composed of unknown coefficients  $b_i^0$ . In order for the problem to have a non-zero solution, the main determinant of system (10) must be equal to zero.

$$\left\|\boldsymbol{\omega}_{1}^{2}\right\| = 0 \tag{11}$$

Equation (11) is an algebraic equation of the n-the order with respect to  $\omega_1^2$  is and although its solution is difficult, it can be calculated in any approximation with the help of computer programs. But in engineering calculations, they are satisfied with the first approximation, that is,  $\omega_1^2$  is determined from the Equation (12):

$$\int_{0}^{a} \gamma(x) \varphi_{1}(x) dx = 0$$
(12)

or from the following one:

$$\int_{0}^{a} \left[ f_{1}(x) \frac{d^{4} \varphi_{i}}{dx^{4}} + 2f'(x) \frac{d^{3} \varphi_{i}}{dx^{3}} + f''(x) \frac{d^{2} \varphi_{i}}{dx^{2}} \right] \varphi_{1} dx + \frac{1}{R_{1}(x)} V_{1}(x) - \omega_{1}^{2} \int_{0}^{a} \left[ \overline{R_{1}(x)} + \overline{\rho} \Psi_{1}(x) \right] \varphi_{1}^{2}(x) dx = 0$$
(13)

If we use the first approximation, we get the expression of  $\omega_1^2$  from Equation (13):

Here, we get:

$$\omega_{l}^{2} = \frac{\int_{0}^{a} \left[ f_{1}(x) \frac{d^{4} \varphi_{l}}{dx^{4}} + 2f_{1}'(x) \frac{d^{3} \varphi_{l}}{dx^{3}} + f_{1}''(x) \frac{d^{2} \varphi_{l}}{dx^{2}} \right] \varphi_{l}(x) dx}{\int_{0}^{a} \left[ \overline{R_{2}(x)} + \overline{\rho} \psi_{1}(x) \right] \varphi_{l}^{2}(x) dx} + \frac{\int_{0}^{a} \left[ \overline{R_{1}(x)} \varphi_{l}(x) \right] \varphi_{l}(x) dx}{\int_{0}^{a} \left[ \overline{R_{2}(x)} + \overline{\rho} \psi_{1}(x) \right] \varphi_{l}^{2}(x) dx}$$
(14)

If the rectangular plate is homogeneous along the thickness and length and located on a viscoelastic basis, then Equation (14) becomes:

$$\omega_0^2 = \frac{\int_0^a \left[ \left( \frac{d^4 \varphi_1}{dx^4} \right) \varphi_1(x) + \overline{R_1(x)} \varphi_1(x) \right] \varphi_1(x) dx}{\int_0^a \left[ \overline{R_2(x)} + \overline{\rho} \right] \varphi_1^2(x) dx}$$
(15)

We use the following values of functions for calculation:

$$\varphi_{1}(x) = \sin(\pi x a^{-1}); \quad R_{1}(x) = R_{1}^{0}(1 + \lambda_{1} x a^{-1}) 
R_{2}(x) = R_{2}^{0}(1 + \chi_{1} x a^{-1}); \quad f_{1}(x) = 1 + \delta_{1} x a^{-1} 
\Psi_{1}(x) = 1 + \mu_{1} x a^{-1}; \quad \chi_{1} \in [0,1]$$

$$\delta_{1} \in [0,1]; \quad \mu_{1} \in [0,1]$$
(16)

If we substitute Equations (16) in (15), we get the following:

$$\omega_0^2 = \frac{\int_a^1 \left[ \left( \frac{\pi}{a} \right)^4 + R_1^0 \left( K_1^0 \right)^{-1} \left( 1 + \chi_1 \beta \right) \right] \sin^2 \pi \beta d\beta}{\left( K_1^0 \right)^{-1} \int_0^1 \left[ R_2^0 \left( 1 + \chi_1 \beta \right) + \rho \right] \sin^2 \pi \beta d\beta}$$
(17)

where,  $\beta = xa^{-1}$  substitution was made.

If the rectangular plate is homogeneous and the resistance of the external environment is taken into account, then Equation (17) can be written as follows:

$$\omega_0^2 = \frac{\left(\frac{\pi}{a}\right)^4 + R_1^0 \left(K_1^0\right)^{-1} \left(1 + 0, 5\lambda_1\right)}{\left(K_1^0\right)^{-1} \left[R_2^0 \left(1 + 0, 5\chi_1\right) + \rho_0\right]}$$
(18)

If the rectangular plate is heterogeneous, then Equation (17) becomes the following:

$$\omega_{1}^{2} = \frac{\left(1+0,5\delta_{1}\right)\left(\frac{\pi}{a}\right)^{4} + R_{1}^{0}\left(K_{1}^{0}\right)^{-1}\left(1+0,5\lambda_{1}\right)}{R_{2}^{0}\left(K_{1}^{0}\right)^{-1}\left(1+0,5\lambda_{1}\right) + \rho_{0}\left(1+0,5\mu_{1}\right)}$$
(19)

After simple substitutions, we write Equation (19) as follows:

$$\omega_2^2 = \frac{p_1 \left(\frac{\pi}{a}\right)^4 + R_1^0 \left(K_1^0\right)^{-1}}{\left(K_1^0\right)^{-1} \left(R_2^0 + \rho_0 p_2\right)}$$
(20)

Here the following substitutions have been made:

$$p_1 = \frac{1 + 0.5\delta_1}{1 + 0.5\chi_1}; \quad p_2 = \frac{1 + 0.5\mu_1}{1 + 0.5\chi_1} \tag{21}$$

If the plate is located on a viscoelastic basis and  $R_2^0 = 0$ , then Equation (20) can be written as follows:

$$\omega_3^2 = \frac{p_1 \left(\frac{\pi}{a}\right)^4 + R_1^0 \left(K_1^0\right)^{-1}}{\rho_0 p_2 \left(K_1^0\right)^{-1}}$$
(22)

If we divide Equations (20) and (22) side by side, we get the value of the dimensionless frequency:

$$\overline{\omega}^{2} = \frac{p_{1}\left(\frac{\pi}{a}\right)^{4} + R_{1}^{0}\left(K_{1}^{0}\right)^{-1}}{\left(K_{1}^{0}\right)^{-1}\left(R_{2}^{0} + \rho_{0}p_{2}\right)} : \frac{p_{1}\left(\frac{\pi}{a}\right)^{4} + R_{1}^{0}\left(K_{1}^{0}\right)^{-1}}{\rho_{0}p_{2}\left(K_{1}^{0}\right)^{-1}} \quad (23)$$

or

$$\overline{\omega}^{2} = \frac{\rho_{0}p_{2}}{R_{2}^{0} + \rho_{0}p_{2}} = \frac{1}{1 + R_{2}^{0} \left(\rho_{0}p_{2}\right)^{-1}}$$
(24)

If we substitute  $r = R_2 (\rho_0 p_2)^{-1}$ , then we write Equation (24) as follows:

$$\overline{\omega}^{-2} = \frac{1}{1+r}, (r > 0)$$
 (25)

The calculation results are given in Table 1 and Figure 1.

Table 1. Calculated values of dimensionless frequency



Figure 1. Dependence between dimensionless frequency inhomogeneity and the parameter characterizing the base resistance

If the resistance of the external environment is not taken into account, Equation (22) becomes as follows:

$$\omega_0^2 = \frac{K_1^0 p_1 \left(\frac{\pi}{a}\right)^4}{\rho_0 p_2}$$
(26)

From here we get the following expression for the dimensionless frequency value:

$$\overline{\omega}_0^2 = \frac{p_1}{p_2} = \frac{1 + 0.5\delta_1}{1 + 0.5\mu_1} \tag{27}$$

The result of calculation is given in Table 2. As it is seen from the graphs in Figure 2, the heterogeneity of the plate and parameters of a heterogeneous viscoelastic basis in fluency seriously upon a frequency value.

Table 2. The value of a dimensionless frequency depending on parameters characterizing heterogeneity

|            |           | $\delta_1 = 0$          | $\delta_1 = 0.2$      |                         | $\delta_1 = 0.4$ |                         |              |
|------------|-----------|-------------------------|-----------------------|-------------------------|------------------|-------------------------|--------------|
|            | $\mu_{l}$ | $\overline{\omega}_0^2$ | $\mu_{l}$             | $\overline{\omega}_0^2$ | $\mu_{l}$        | $\overline{\omega}_0^2$ |              |
|            | 0         | 1                       | 0                     | 1.1                     | 0                | 1.2                     |              |
|            | 0.2       | 0.9090                  | 0.2                   | 1                       | 0.2              | 1.090                   |              |
|            | 0.4       | 0.8333                  | 0.4                   | 0.9166                  | 0.4              | 1                       |              |
|            | 0.6       | 0.7690                  | 0.6                   | 0.8461                  | 0.6              | 0.9230                  |              |
|            | 0.8       | 0.7141                  | 0.8                   | 0.7857                  | 0.8              | 0.8571                  |              |
|            | 1         | 0.6666                  | 1                     | 0.7333                  | 1                | 0.8                     |              |
|            | δ         | 1 = 0.6                 | $\delta_{\mathrm{I}}$ | = 0.8                   | 0                | $\delta_1 = 1$          |              |
|            | $\mu_{l}$ | $\overline{\omega}_0^2$ | $\mu_{l}$             | $\overline{\omega}_0^2$ | $\mu_{l}$        | $\overline{\omega}_0^2$ |              |
|            | 0         | 1.3                     | 0                     | 1.4                     | 0                | 1.5                     |              |
|            | 0.2       | 1.1818                  | 0.2                   | 1.2727                  | 0.2              | 1.3636                  |              |
|            | 0.4       | 1.0831                  | 0.4                   | 1.1666                  | 0.4              | 1.25                    |              |
|            | 0.6       | 1                       | 0.6                   | 1.0769                  | 0.6              | 1.153                   |              |
|            | 0.8       | 0.9285                  | 0.8                   | 1                       | 0.8              | 1.0714                  |              |
|            | 1         | 0.8666                  | 1                     | 0.9333                  | 1                | 1                       |              |
| $\omega^2$ |           |                         |                       |                         |                  | $\delta_1$              | = 0<br>= 0.2 |
| 1.0        |           |                         |                       |                         | _                | $\delta_3$              | = 0.4        |
| · · · ·    |           |                         |                       |                         | -                | $ \delta_4$             | = 0.6        |
| 14         |           |                         |                       |                         | _                | $\delta_5$              | = 0.8        |
|            |           |                         | ••••                  |                         |                  | $\ldots \delta_6$       | =1.0         |
|            |           |                         |                       |                         |                  |                         |              |
| 1.2        | -         |                         |                       |                         |                  |                         |              |
|            | -         |                         | -                     | _                       |                  |                         |              |
|            | _         |                         |                       |                         |                  |                         |              |
| 1          |           |                         | _                     |                         |                  |                         |              |
|            |           |                         |                       |                         |                  |                         |              |
| 0.4        |           |                         |                       | *******                 |                  |                         |              |
|            |           |                         |                       |                         |                  |                         | ********     |
| 0.6        |           |                         |                       |                         |                  |                         |              |
| 0.0        | 0.2       | 0                       | .4                    | 0.                      | 6                | 0.8                     | $\mu_1 = 1$  |
|            |           |                         |                       |                         |                  |                         |              |

Figure 2. Dependence between the dimensionless frequency and the parameter characterizing heterogeneity

### 4. FREQUENCY ANALYSIS OF A PLATE USING FINITE ELEMENT METHOD

The frequency analysis of the plate was performed in the SolidWorks program. For this purpose, plates made of AISI 1020 material with a ratio of measures of length a/b = 0.5, a/b = 1, a/b = 1.5 thickness h = 10 mm were chosen. The elastic modulus of the material is  $E = 200000 \text{ N/mm}^2$ , the Poisson's ratio is v = 0.29, its density is  $\rho = 7900 \text{ kg/m}^3$ , and sliding elastic modulus is  $G = 77000 \text{ N/mm}^2$ . The plate was analyzed at 6 frequency values. The analysis results are shown in Figure 3.

In Figure 3a, the mode does not vary in frequency for the first three voltage states. The maximum frequency in mode 4 is distributed along the length of the plate, in mode 5 along the width, and in mode 6 in a circular shape along the width. In Figure 3b, there is no change in the position of the plate for modes 1 and 3. In modes 4 and 5, the frequency distribution is observed in a linear form in the outer zones in the longitudinal direction, and in mode 6, it is observed in a curved line form in the corner points.



Figure 3. Simulation results on a frequency analysis a) a/b = 0.5; b) a/b = 1; c) a/b = 1.5

| <i>a / b</i> = 0.5 |           |           |        |  |  |  |  |  |
|--------------------|-----------|-----------|--------|--|--|--|--|--|
| Mode               | Frequency | Frequency | Period |  |  |  |  |  |
| No                 | (Rad/Sec) | (Hz)      | (sec)  |  |  |  |  |  |
| 1                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 2                  | 0.007944  | 0.0012643 | 790.94 |  |  |  |  |  |
| 3                  | 0.0088798 | 0.0014133 | 707.58 |  |  |  |  |  |
| 4                  | 0.011838  | 0.0018841 | 530.76 |  |  |  |  |  |
| 5                  | 0.015773  | 0.0025104 | 398.35 |  |  |  |  |  |
| 6                  | 0.016048  | 0.0025541 | 391.53 |  |  |  |  |  |
| <i>a / b</i> = 1   |           |           |        |  |  |  |  |  |
| 1                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 2                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 3                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 4                  | 0.0053268 | 0.008477  | 1179.5 |  |  |  |  |  |
| 5                  | 0.001104  | 0.001752  | 570.49 |  |  |  |  |  |
| 6                  | 0.01262   | 0.0020086 | 497.86 |  |  |  |  |  |
| <i>a / b</i> = 1.5 |           |           |        |  |  |  |  |  |
| 1                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 2                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 3                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 4                  | 0         | 0         | 1e+32  |  |  |  |  |  |
| 5                  | 0.011436  | 0.001820  | 549.42 |  |  |  |  |  |
| 6                  | 0.014157  | 0.002253  | 443.83 |  |  |  |  |  |

Table 3. Frequency results

In Figure 3c, mode 1 and 3 are not variable in the state of the plate. In mode 4, the maximum values are formed in a linear form in the outer zones along the length, and in mode 5, the maximum values are formed in the outer zones along the width. In Mode 6, the value is distributed from the center in the form of a curved line. The maximum value is formed in the corner points of the plate. The frequency value results for all three cases are given in Table 3.

In the ratio a/b = 0.5, the frequency of the first mode is equal to zero. When passing from mode-2 and mode-6, the frequency increases by 50%. The frequency of the first three modes is equal to 0 in the plate with the size ratio a/b = 1. Between mode 4 and mode 6, there is a 76% drop in frequency values.

When a/b = 1.5, the frequency values for the first 4 modes are equal to zero. When passing from mode 5 to mode 6, the frequency increases by 19%. In all three cases, the maximum frequency was observed in the plate with a/b = 0.5, and the lowest frequency was observed in the plate with a/b = 1.5. In a plate with a/b = 0.5, the frequency increases as the number of modes increases. In plates with a/b = 1 and a/b = 1.5, frequency values decrease as the number of modes increases.

#### 5. CONCLUSION

The structure studied in this paper is considered a special-purpose construction element because it is applied in special areas of industry. The fact that the heterogeneity of a rectangular plate depends on one coordinate (on the direction of its largest dimension) allows it to oscillate in a conical or cylindrical shape. Such construction elements are thin-walled, so they are membrane-like, and they are widely used in the drainage of marshes (free transport of marsh gas), building and assembly jobs (initial increase of the road bed) on saline soils or deserts. The following results were obtained in the paper:

1. Cylindrical free oscillations of a rectangular plate located on a heterogeneous viscoelastic base were studied in this work. In order to solve the problem, an ordinary differential equation of the fourth order with variable coefficients was constructed.

The solution of the obtained equation was chosen in such a way as to ensure the cylindrical oscillation of the plate, at the same time, a homogeneous boundary condition was established for the solution of the problem.
 An error function was constructed and used to ensure the accuracy of the numerical solution of the differential equation of the cylindrical free oscillation.

4. A cylindrical oscillation of the plate was analyzed according to the change of the numerical values of the physical and mechanical characteristics of the studied structural element and it was determined that those characteristics change the circular frequency of the oscillation (the frequency decreases as the heterogeneity and viscosity of the medium increases).

5. As can be seen from the obtained equations, if the heterogeneity of the material and the resistance of the external environment are not taken into account, then there are serious errors in engineering calculations.

6. At the same time, it was determined that the shape of the oscillations depends on the fastening conditions of the contours of the plate (jointed fastening two parallel edges correspond to a cylindrical oscillation, fastening three sides corresponds to a conical oscillation, jointed fastening from four sides corresponds to a spherical oscillation).

7. Finally, a solution methodology for free-oscillating engineering devices in a resistive environment is offered. 8. Also, a frequency analysis was performed using the finite element method in the "Solidworks" program of the plate with a ratio of a/b = 0.5, a/b = 1, a/b = 1.5. It was determined that the smallest value of the frequency is observed in the plate with the size a/b = 1.5, and the largest value is observed in the plate with a/b = 0.5. The comparative analysis of the results shows that in the frequency calculations, the heterogeneity of the material and the resistance of the external environment have a serious effect on frequency values. The obtained results can be used in calculation of frequency-amplitude characteristics of rectangular plates used in construction and shipbuilding industry.

#### NOMENCLATURES

#### Symbols / Parameters

 $E_1^0$ ,  $E_2^0$ : elasticity modulus

 $G^0$ : elasticity modulus at sliding

 $\rho_0$ : density

 $f_1(x)$ : function shows heterogeneity of the rectangular plate along the length

 $\Psi_1(x)$ : continuous function characterizing density change

- $v_1^0$ ,  $v_2^0$ : Poisson's coefficients
- b: the thickness of the rectangular orthotropic plate
- $\omega_{l}$ : frequency
- $W_1$ : bending deformation

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