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PLANAR STABILITY OF AN ELASTIC, PLASTIC BEAM DIFFERENTLY RESISTING TO TENSION AND COMPRESSION

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Abstract- Nowadays, beams, boards and coatings with new complex properties are widely used in many branches of mechanical engineering and construction. The calculation and analysis of the amplitude characteristics of stability, strength and frequency of a structural element with these properties lead both to significant difficulties in mathematical terms and to the analysis of the results obtained, if ignored, serious errors may be made. Taking these into account, it becomes necessary to build mathematical models characterizing the real properties of the material when using a structural element made of new materials and establishing effective physical connections. There are materials in which the tensile strain diagrams characterizing their properties are diverse in tension-compression and torsion. Such materials include ceramics, some types of copper and cast iron, polymers, and composite materials. The mechanical and physical properties of these materials become strictly dependent on hydrostatic pressure. For materials with the above-mentioned specific property, classical elasticity and elastoplasticity cannot be considered under the conditions accepted by the theory of plasticity. In this paper, a problem of planar stability of an elastic, plastic plane differently resisting to tension and compression in pure bending is solved. It is assumed that the cross-section of the beam has one symmetry axis, under the action of concentrated moments applied at the ends of the bar is subjected to pure bending, and bending happens in the symmetry plane of the beam. Using the state of a neutral axis, absence of longitudinal force, continuity conditions for tension and compression, we determine the boundary of elastic and plastic domains. The equations of the loss of planar stability obtained for the classic case are reduced to the loss of planar stability for various modulus ideal elastic and plastic beams. The expressions of hardness for different modulus materials are obtained and are associated with critical moment and critical length. Expressions for calculating critical parameters for an ideally elastic, plastic beam with a rectangular cross section are obtained.

Keywords: Tension, Compression, Ideally Plastic, Beam, Stability.

1. INTRODUCTION

Beams made of different materials are widely used in the construction of modern engineering complexes, bridges of various purposes, in the construction of overpasses, in machine building and in other fields. In order to reduce the weight of the construction parts and increase the quality indicators, it is necessary to work on an accurate calculation methodology. In modern times there are materials, whose mechanical properties do not obey the laws of the classic theory of elasticity and plasticity, have differently resistance to tension and compression.

Functional degree materials, composite, polymer materials, some kinds of cast iron are among these materials. To this end, the article is devoted to solving the problem of stability of the flat shape of ideally elastic plastic beams. Here, the hypothesis was accepted that the beam material has different tensile and compressive resistance.

In calculation of planar stability in engineering works, within the classic theory the hypothesis suggested by S.P. Timoshenko is used. It is assumed that in the loss of planar stability, the bending and torsional hardness of the beam changes as a hardness of the elastic beam in longitudinal bending after elasticity limit [1].

But it is shown in [6] that the above accepted hypothesis leads to a decrease in the value of the hardness. It is clear that the above solution is not acceptable for beams differently resisting to tension and compression.

In [2], [3] the stability theory of initial stress elastic, plastic structural elements has been built and a number of practical importance problems have been proved N.M. Timoshenko and L.A. Tolokonnikov have researched relations between stresses and deformations for materials differently resisting to tension and compression [4].

In his monograph, O.B. Kochin has shown methods for calculating structural elements made of inhomogeneous materials. Here the structural elements made of different modulus materials are not considered and the resistance of the environment is not taken into account [5].

In [6] and [7], the problem of stability of beams made of linearly strengthened elastic plastic material has been solved with resistance of the environment taken into account.

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The paper [8] has been devoted to the bending stability of beams behaving as elastic and plastic and with a rectangular cross section. Here three different functions have been obtained for special cases in bending of the beam under typical loading.

In [9], [10] two functions have been obtained due to bending and shift for structural elements made of two different composite materials. Analytic and numerical solutions corresponding to the material features have been made. The bending resistance of elastic and ideal plastic beams, one end of which is rigidly fixed and the other is subjected to step loads, has also been studied. It is established that the compliance of the deformation of the support with the law of change depends on the numerical value of step loads.

2. THE PROBLEM STATEMENT AND SOLUTION

Assume that E^+, E^- are elasticity modules v^+, v^- are Poisson ratios and σ_s^+, σ_s^- are yield points of a beam differently resisting to tension and compression (Figure 1).

The coordinate axes are chosen as follows: The axis Oz is directed along the central line of undeformed beam, the axes x and y are located on the plane of the last section of the beam (Figure 2).

It is assumed that the beam was fixed so that the last sections can turn freely, they can not rotate around the axis z i.e., the following conditions must be fulfilled:

$$z = 0 \text{ and } z = l \text{, for } V_0 = 0 \text{ ; } V_0' = 0. \tag{1}$$

where, before the stability loss, the hypothesis of planar sections is accepted. The following expressions are valid under the accepted conditions:

$$\varepsilon_1 = e_0 - z\wp_0; \ \varepsilon_2 = \varepsilon_3 = -\frac{1}{2}\varepsilon_1 \tag{2}$$

where, ε_1 is a relative deformation of the middle line before the loss of stability, $\wp_0 = \dfrac{d^2W}{dx^2}$ is a curvature, and V_0 is a curve.

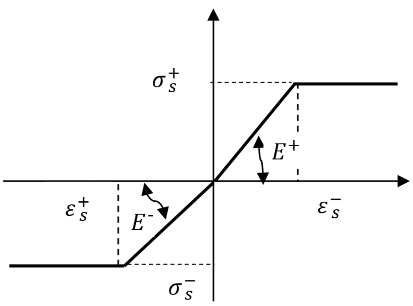

Figure 1. Stress-strain diagram for different modulus ideally elastic, plastic materials

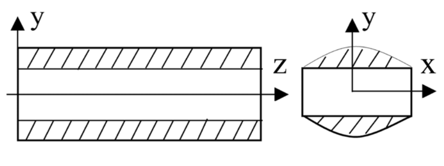

Figure 2. Boundaries of elastic plastic domains at tension and compression

The following domains can exist in the section before the loss of stability: at tension S_1^+, at compression S_3^- elastic domains, S_1^+, S_3^- plastic domains.

- Equilibrium conditions: The condition of the absence of longitudinal force:

$$\iint_{s'} \sigma dxdy = 0 \tag{3}$$

The condition of equality of bending moments:

$$\iint_{s'} x\sigma dxdy = M \tag{4}$$

where, S' is the cross-sectional area.

The boundaries of elastic and plastic domains can be determined by adding the state of the neutral axis, continuity conditions and the equation of the neutral axis:

Taking these into account we can write Equation (5). The equation of the neutral axis

$$e_0 - y_0 \cdot \rho_0 = 0 \tag{5}$$

- Continuity conditions:

$$\begin{aligned} &E^+ \left(e_0 - z \cdot \rho_0\right) = \sigma_s^+; \ z \in S_1^+ \\ &E^- \left(e_0 - z \cdot \rho_0\right) = \sigma_s^-; \ z \in S_2^- \end{aligned} \tag{6}$$

At first, we study the expressions (3) and (4):

$$\begin{aligned} &E^+ \iint_{S_1^+} \left(e_0 - y\wp_0\right) ds + \sigma_s^+ \iint_{S_2^+} ds + \\ &+E^- \iint_{S_3^-} \left(e_0 - y\wp_0\right) ds + \sigma_s^- \iint_{S_4^-} ds = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &M = E^+ \iint_{S_1^+} y\left(e_0 - y\wp_0\right) ds + \sigma_s^+ \iint_{S_2^-} yds + \\ &+E^- \iint_{S_3^-} y\left(e_0 - y\wp_0\right) ds + \sigma_s^- \iint_{S_4^-} yds \end{aligned} \tag{8}$$

$$\begin{aligned} &\iint_{S_1^+} \left(e_0 - y\wp_0\right) ds + \varepsilon_s^+ \iint_{S_2^+} ds + \\ &+\alpha \left[\iint_{S_3^-} \left(e_0 - y\wp_0\right) ds + \varepsilon_s^- \iint_{S_4^-} ds \right] = 0 \end{aligned} \tag{9}$$

$$\begin{aligned} &\frac{M}{E^+} = \iint_{S_1^+} y\left(e_0 - y\wp_0\right) ds + \varepsilon_s^+ \iint_{S_2^+} yds + \\ &+\alpha \left[\iint_{S_3^-} y\left(e_0 - y\wp_0\right) ds + \varepsilon_s^- \iint_{S_4^+} yds + \right] \end{aligned} \tag{10}$$

$$e_0 = y\wp_0 \tag{11}$$

where, y_0 is the boundary of the neutral axis from the cross section of the beam.

Having written the expression (11) in the equations (9) and (10), we reduce e_0.

$$\begin{aligned} &\wp_0 \iint_{S_1^+} \left(y_0 - y\right) ds + \varepsilon_s^+ \iint_{S_2^+} ds + \\ &+\alpha \left[\wp_0 \iint_{S_3^-} \left(y_0 - y\right) ds + \varepsilon_s^- \iint_{S_4^-} ds \right] = 0 \end{aligned} \tag{12}$$

$$\iint_{S_1^+} y\left(y_0-y\right)ds+\varepsilon_s^+\iint_{S_2^+} yds+$$
$$+\alpha\left[\wp_0\iint_{S_3^-} y\left(y_0-y\right)ds+\varepsilon_s^-\iint_{S_4^-} yds\right]=\frac{M}{E^+\wp_0} \tag{13}$$

From the equation (13) we can obtain:

$$\wp_0=-\frac{\varepsilon_S^+\iint_{S_2} ds+\alpha\varepsilon_s^-\iint_{S_4^-} ds}{\iint_{S_1^+}\left(y_0-y\right)ds+\alpha\iint_{S_3^-}\left(y_0-y\right)ds} \tag{14}$$

Writing the expression (14) in (13), we can obtain the following expression:

$$\frac{M}{E^+}=-\frac{\varepsilon_S^+\iint_{S_2^+} ds+\alpha\varepsilon_s^-\iint_{S_4^-} ds}{\iint_{S_1^+}\left(y_0-y\right)ds+\alpha\iint_{S_3^-}\left(y_0-y\right)ds}\times$$
$$\times\left(\iint_{S_1^+} A_1^+ds+\alpha\iint_{S_3^-} A_3^-ds\right)+ \tag{15}$$
$$+\alpha\left(\varepsilon_S^+\iint_{S_2^+} yds+\varepsilon_S^-\iint_{S_4^-} yds\right)$$

where, $A=y\left(y_0-y\right)$.

Thus, when the parameters $\sigma_S^+,\sigma_S^-,E^+,E^-$ are characterizing the features of the material and the form of the section are known, we can determine the state before the loss of stability.

It should be noted that when the property of the material is the same as in the figure, we can accept that the bound arise of the possible plastic domains do not change in the state after the loss of stability [1, 2].

In a critical situation, when the stiffness value in relation to the x axis is less than in the y axis, a loss of plane stability occurs [1]. Considering that with loss of stability, the value of the pair creating the bend remains constant, we can write the equilibrium conditions as follows:

$$\iint_S \delta\sigma ds=0;\ \iint_S x\delta\sigma ds=0$$
$$\iint_S y\delta\sigma ds=\psi M;\ \iint_S yd\tau ds=-\frac{du}{dz}M \tag{16}$$

where, ψ is a turning angle of the cross-section around the axis z. The integration in (16) is done in the section $z=\text{const}$. It is known that the loss of planar stability is characterized in the classic theory of elasticity by system of Equations (17) [11, 12]:

$$B_0\frac{d^2u}{dz^2}=\psi M$$
$$C_0\frac{d\psi}{dz}=-\frac{du}{dz}M \tag{17}$$

where, B_0 is a bending hardness, C_0 is a torsional hardness. Now we show the rule how the Equation (16) passes to the Equation (17), in other words determine the B^* and C^* hardness for the beam differently resisting to tension and compression. Here we should take into account that in plastic domains the stress variation is zero [2].

$$\iint_{S_1^+} C(y_0)\left(e-C(y_0)\cdot\rho\right)ds+\iint_{S_3^-} C(y_0)\left(e-C(y_0)\cdot\rho\right)ds+$$
$$+\iint_{S_2^+} ds+\iint_{S_1^-} ds \tag{18}$$

$$C^*=\iint_{S_1^+}\left(y\frac{\partial\phi}{\partial x}-x_1\frac{\partial\phi}{\partial y}+x^2+y^2\rho\right)ds+$$
$$+\iint_{S_3^-}\left(y\frac{\partial\phi}{\partial x}-x\frac{\partial\phi}{\partial x_2}-x^2+y^2\right)ds+ \tag{19}$$
$$+\iint_{S_2^+} ds+\iint_{S_4^-} ds$$

where,

$$C(y_0)=0.5\left[y_0+(y_0^2-A)^{\frac{1}{2}}\right]$$

The ϕ is a function included in the hardness expression is chosen as in [1].

Thus, the problem considered after the determination of B^* and C^* hardness is reduced to the solution of the following system of equations:

$$B_0B^*\frac{d^2u}{dz^2}=\psi M$$
$$C_0C^*\frac{d\phi}{dz}=-M\frac{du}{dz} \tag{20}$$

From the system (20) we obtain the following condition:

$$\bar{M}_k=\sqrt{B^*C^*}$$
$$\bar{l}_k=\sqrt{B^*C^*} \tag{21}$$

or

$$\bar{M}_k=M_kM_0^{-1};\ \bar{l}_k=l_kl_0^{-1} \tag{22}$$

where, $M_k;l_k$ are critic moment and critic length of the elastic beam, respectively.

Let us show the solution obtained above for elastic and plastic beam with a rectangular cross-section and differently resisting to tension and compression.

Assume that before the loss of stability at first the plastic zone is formed in one zone. Since $\left(\sigma^-<\sigma^+\right)$ this zone will happen in the compressed zone of the beam (Figure 3).

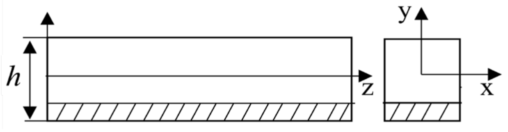

Figure 3. Formation of plastic deformation zone at compression

The equilibrium equations in the state before the loss of stability will be written as follows:

$$
\begin{aligned}
&-\sigma_S^- \int_{-a}^{+a}\int_{-h}^{\eta_1} dxdy + E^- \int_{-a}^{+a}\int_{\eta_1}^{\eta_0} \left(e_0 - \wp_0 x\right) dxdy + \\
&+E^+ \int_{-a}^{+a}\int_{\eta_0}^{+h} \left(e_0 - \wp_0 x\right) dxdy = 0
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&-\sigma_S^- \int_{-a}^{+a}\int_{-h}^{\eta_1} dxdy + E^- \int_{-a}^{+a}\int_{\eta_1}^{\eta_0} \left(e_0 - \wp_0 x\right) dxdy + \\
&+E^+ \int_{-a}^{+a}\int_{\eta_0}^{+h} \left(e_0 - \wp_0 x\right) dxdy = M
\end{aligned}
\tag{24}
$$

where, η_0 is a boundary of elastic, plastic deformation of the neutral axis η_1 is an elastic, plastic deformation boundary of the compression zone. After simple transformations we obtain the following equations:

$$
-2a\varepsilon_S^{-1}\left(\eta_0 - \eta_1\right) + ae_0 + 2ahe_0\left(h - \eta_0\right) = 0 \tag{25}
$$

The Equation (25) takes the following form:

$$
\begin{aligned}
&-\frac{\wp_0 E^-}{3}\left(\eta_0^3 - \eta_1^3\right) - \frac{E^+}{3}\left(h^3 - \eta_0^3\right) = M \\
&-\frac{\wp_0}{3}\left[\eta_0^3 - \eta_1^3 + \alpha\left(h^3 - \eta_0^3\right)\right] = M
\end{aligned}
\tag{26}
$$

If plastic deformation is formed during stretching, the equilibrium equations are written as follows (Figure 4).

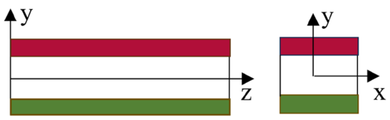

Figure 4. Plastic deformation formation at tension

$$
\begin{aligned}
&-\sigma_S^- \int_{-a}^{+a}\int_{-h}^{\eta_1} dxdy + E^- \int_{-a}^{+a}\int_{\eta_1}^{\eta_0} \left(e_0 - \rho_0 x\right) dxdy + \\
&+E^+ \int_{-a}^{+a}\int_{\eta_0}^{\eta_2} \left(e_0 - \rho_0 x\right) dxdy + \\
&+\sigma_S^+ \int_{-a}^{+a}\int_{\eta_2}^{+h} dxdy = 0
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&-\sigma_S^- \int_{-a}^{+a}\int_{-h}^{\eta_1} xdxdy + E^- \int_{-a}^{+a}\int_{\eta_1}^{\eta_0} x\left(e_0 - \wp_0 x\right) dxdy + \\
&+E^+ \int_{-a}^{+a}\int_{\eta_0}^{\eta_2} \left(e_0 - \wp_0 x\right) xdxdy + \\
&+\sigma_S^+ \int_{-a}^{+a}\int_{\eta_2}^{+h} xdxdy = M
\end{aligned}
\tag{28}
$$

After some simple transformations we obtain:

$$
\begin{aligned}
&-\sigma_S^- 2a\left(\eta_1 + h\right) + 2aE^- e_0\left(\eta_0 + \eta_1\right) + \\
&+\sigma_S^+ 2a\left(h - \eta_2\right) + E^+ 2a\left(\eta_0 - \eta_2\right)e_0 = 0 \\
&-\frac{\wp_0 E^-}{3}\left(\eta_0^3 - \eta_1^3\right) - \frac{E^+}{3}\wp_0\left(\eta_2^2 - \eta_0^3\right)2a = M
\end{aligned}
\tag{29}
$$

In pure quantities:

$$
\left(\overline{\varepsilon}_0 = \frac{\varepsilon_0}{\varepsilon_S^-}; \rho_1 = \frac{\eta_1}{h}; \rho_2 = \frac{\eta_2}{h}; \overline{\rho}_0 = \frac{\eta_0}{h}\right)
$$

$$
\begin{aligned}
&M = \frac{E^- ah^3}{3}\left[\alpha\left(1 + \rho_0^3\right) + \rho_0^3\right] \\
&M = \frac{2E^- ah^3}{3}\left[\left(\rho_1^3 - \rho_0^3\right) - h\left(\rho_2^3 - \rho_0^3\right)\right]
\end{aligned}
\tag{30}
$$

From the first equation we obtain the dependence between $\eta_0, \eta_1, \eta_2, \varepsilon_0$.

We determine the hardness in bending as follows:

$$
\begin{aligned}
B^* = \frac{1}{2}\Big\{&\frac{1}{n}\left[\left(1 - \lambda^+\right)\left(1 + \rho_1\right) + \rho_0 - \rho_1\right] + \\
&+\rho_2 - \rho_0 + \left(1 - \lambda^-\right)\left(1 - \rho_2\right)\Big\}
\end{aligned}
\tag{31}
$$

If the material of the beam is linear, of different modulus, (31) takes the following value:

$$
B^* = \frac{1}{2}\left[\frac{1}{n}\left(1 + \rho_0\right) + 1 - \rho_0\right] \tag{32}
$$

We can calculate the hardness C^* in torsion by the given methodology. The classic solution is obtained for $E^+ = E^-$

As a result of calculation, the discharge effect is not taken into account when the stability is lost.

3. COMPARISON OF RESULTS

The issues closest to the one considered in the presented article are considered in [1], [2, 3], [4, 5] by its authors. However, in the questions studied by these authors, the hypothesis that the structural elements have different tensile and compressive resistance was not taken into account, for this reason, the actual stress-strain state of structural elements differs in accuracy from the results obtained by the above-mentioned authors, the difference between which is 9.5%.

The reported values of the dimensionless critical moment of measurement and the critical length, depending on the values of the flexural and torsional stiffness expressions, are shown in Table 1.

Table 1. Dimensionless values of critical moment and critical length ($\overline{M_k} = \overline{l_k} = \sqrt{B^* C^*}$)

$\alpha = 0.5$			
ρ_0	B^*	C^*	$\overline{M_k} = \overline{l_k}$
0.1	1.550	0.954	1.216
0.2	1.600	0.968	1.244
0.3	1.650	0.980	1.271
0.4	1.700	0.988	1.295
0.5	1.750	0.995	1.319
0.6	1.800	1.000	1.341
0.7	1.850	1.002	1.361
0.8	1.900	1.003	1.380

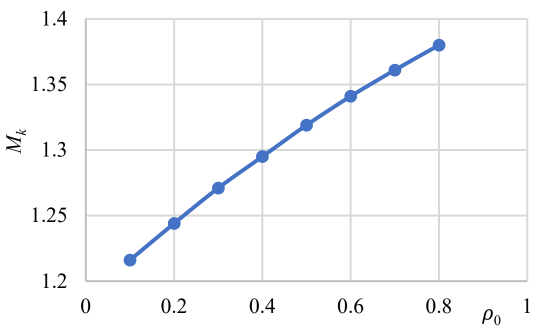

Figure 5. The relationship between the dimensionless value of the critical moment and the boundary of the neutral axis in the cross section of the beam

As can be seen from Table 1 and Figure 5, ρ_0 is increases, the effect of different modularity on critical parameters increases. As can be seen from the results obtained, serious errors can be made if you do not take into account the property of the materials mentioned in the durability reports to exhibit various tensile and compressive resistances.

Let's use the equations and formulas obtained in article and consider their application to solving a simple problem.

Suppose that a beam with a rectangular cross-section undergoes a clean bend, and at some of its magnitude, the stability of the flat shape occurs. The material of the beam is an ideal plastic with the first and second elasticity (Figure 6).

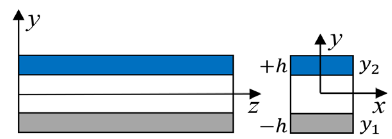

Figure 6. An ideal plastic beam of rectangular cross-section, the material of which has the first and second elasticity.

We believe that the plastic zone and the second elastic region arise in the areas of tension and compression. The bending moment depends on the magnitude of the loss of stability.

$$\sigma_S^+ > \sigma_S^- \tag{33}$$

In this case, the formation of the first plastic zone will occur in the compressed area, and then in the stretched area. The position of the neutral axis is determined from the following condition:

$$e_0 = \wp_0 y_0 \tag{34}$$

The state before the loss of stability is related to the following equations:

$$\begin{aligned} &E_1^+\int_{-a}^{+a}\int_{y_0}^{y_1}\left(e_0-\wp_0 x\right)dxdy+\int_{-a}^{+a}\int_{y_1}^{y_2}\sigma_S^+ dxdy+ \\ &+E_2^+\int_{-a}^{+a}\int_{y_2}^{h}\left(e_0-\wp_0 y\right)dxdy+E_1^-\int_{-a}^{+a}\int_{-h}^{y_3}\left(e_0-\wp_0 y\right)dxdy+ \\ &+\int_{-a}^{+a}\int_{y_3}^{y_4}\sigma_S^- dxdy+E_2^-\int_{-a}^{+a}\int_{y_4}^{y_0}\left(e_0-\wp_0 x\right)dxdy=0 \end{aligned} \tag{35}$$

$$\begin{aligned} &E_1^+\int_{-a}^{+a}\int_{y_0}^{y_1}x\left(e_0-\wp_0 x\right)dxdy+\int_{-a}^{+a}\int_{y_1}^{y_2}x\sigma_S^+ dxdy+ \\ &+E_2^+\int_{-a}^{+a}\int_{y_2}^{h}x\left(e_0-\wp_0 y\right)dxdy+E_1^-\int_{-a}^{+a}\int_{-h}^{y_3}x\left(e_0-\wp_0 y\right)dxdy+ \\ &+\int_{-a}^{+a}\int_{y_3}^{y_4}x\sigma_S^- dxdy+E_2^-\int_{-a}^{+a}\int_{y_4}^{y_0}x\left(e_0-\wp_0 x\right)dxdy=M \end{aligned} \tag{36}$$

Let's consider the following special cases of studying these equations:

3.1. The Rod Material Is Elastic and Has Different Tensile and Compressive Resistance

The equations in this case are reduced to the following form:

$$E_1^+\int_{-a}^{+a}\int_{y_0}^{+h}\left(y_0-y\right)dxdy+E_1^-\int_{-a}^{+a}\int_{-h}^{y_0}\left(y_0-y\right)dxdy=0$$

$$E_1^+\int_{-a}^{+a}\int_{y_0}^{+h}y\left(y_0-y\right)dxdy+E_1^-\int_{-a}^{+a}\int_{-h}^{y_0}y\left(y_0-y\right)dxdy=M\cdot\wp_0^{-1}$$

or

$$\begin{aligned} &y_0\left(h-y_0\right)-\frac{1}{2}\left(h^2-y_0^2\right)+ \\ &+\alpha_1\left[y_0\left(y_0+h\right)-\frac{1}{2}\left(y_0^2-h^2\right)\right]=0 \\ &\frac{y_0}{2}\left(h^2-y_0^2\right)-\frac{1}{3}\left(h^3-y_0^3\right)+ \\ &+\alpha_1\left[\frac{y_0}{2}\left(y_0^2-h\right)-\frac{1}{3}\left(y_0^2+h^3\right)\right]=\frac{M}{2E_1^+a\wp} \end{aligned} \tag{37}$$

Let's take the following substitutions:

$$\alpha_1=\frac{E_1^+}{E_1^-};\rho_0=y_0\cdot h^{-1};\ \bar{M}=\frac{3}{2ah^3E_1^+}$$

The equation of the system (37) reduces to the following form:

$$\begin{cases} \rho_0\left(1-\rho_0\right)-\frac{1}{2}\left(1-\rho_0^2\right)+\alpha_1\left[\rho_0\left(\rho_0\left(\rho_0+1\right)-\frac{1}{2}\left(\rho_0^2-1\right)\right)\right]=0 \\ 3\left[\left(1-\rho_0^2\right)-\frac{1}{3}\left(1-\rho_0^3\right)\right]+\alpha_1\left[\frac{\rho_0}{2}\left(\rho_0^2-1\right)-\frac{1}{3}\left(1+\rho_0^3\right)\right]=\bar{M}\wp_0^{-1} \end{cases} \tag{38}$$

It can be seen from the equation of the system (38) that for the modular scale it becomes $\rho_0=0$.

From the first equation of the system (38), ρ_0 is determined: if we write this value instead in the second Equation (38), we determine the relationship between α_1, $\bar{M}$ and $\bar{\wp}$:

$$\bar{M}=K^*\wp \tag{39}$$

3.2. The Rod Material with Different Tensile and Compressive Resistance Is Elastoplastic

After some substitutions, Equations (38) are written as follows:

$$\wp\left[\rho_0\left(\rho_1-\rho_0\right)-\frac{1}{2}\left(\rho_1^2-\rho_0^2\right)\right]+$$
$$+\varepsilon_s^+\left(1-\rho_1\right)+\frac{\sigma_s^-}{E_1^+}\left(\rho_2+1\right)+\alpha\wp_0\left(\rho_0-\rho_2\right)- \tag{40}$$
$$-\left[\frac{\rho_0}{2}\left(\rho_0^2-\rho_2^2\right)-\frac{1}{2}\left(\rho_0^2-\rho_2^3\right)\right]=0$$

It should be noted that in conclusion, the question of finding the critical moment is brought to the classical problem of elasticity, when, as in the previous question, the mechanical properties of the material are known.

4. CONCLUSIONS

The stability problem for beams made of ideally elastic plastic material and differently resisting to tension and compression was solved. Here, using the condition of absence of longitudinal force, the equality of the bending moment, the boundaries of elastic and plastic domain for the state before the loss of stability were determined. The expression for the curvature with respect to the state of the neutral axis, was derived. Depending on the parameters characterizing the properties of the material of beams with different cross-sections, the relationship equation was obtained to determine the state before the disappearance of the support.

Taking into account the resistance of the viscoelastic base, it is necessary to take into account that the structural element (beam, shell, etc.) offers solutions to the problem of dynamic stability.

For the first time, the basis was laid for the question of vibrations and stability of elastoplastic beams on an inhomogeneous base.

Accepting the theory of the loss of planar stability, we obtained the expressions for a critical moment and length with respect to the hardness values for beams of rectangular cross-section. The obtained expressions can be applied in stability analysis of structural elements made of ideally elastic, plastic materials (composite, high quality materials, some kinds of cast iron).

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