

PERSPECTIVE

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Cultivating optimization intuition in engineering education through multi-methodological problem solving

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Abstract

This paper presents a multi-methodological approach to solving optimization problems within the framework of engineering education. While traditional calculus-based methods focusing on derivatives are the standard pedagogical tool, they often fail to provide students with a deep physical and geometric intuition. We analyze several complex optimization cases, including radical sums, structural trapezoidal geometry, and fundamental mathematical constants, to demonstrate how visual-geometric methods can yield more efficient and insightful solutions. The results suggest that integrating these "short-cut" geometric strategies alongside traditional calculus enhances the analytical capabilities of engineering students and fosters a more robust understanding of mathematical modeling in practical applications.

Keywords Engineering education, Geometric optimization, Mathematical modeling, Pedagogical innovation, Visual-algebraic approach

1 Introduction

The field of engineering constantly demands efficient solutions to optimization problems, ranging from trajectory planning in aerospace to power distribution in electrical grids. Traditionally, pedagogical approaches heavily rely on differential calculus and derivative-based methods. While effective, these analytical tools can sometimes distance the learner from the underlying physical and geometric properties of the problem. This paper proposes a transition toward a "visual-algebraic" approach, where geometric intuition serves as a primary tool for finding extreme values. By examining diverse cases—from radical sums to structural geometry—we illustrate how visual methods provide not only faster results but also a deeper conceptual understanding essential for modern engineering practices [1, 2].

2 Geometric optimization of radical sums

2.1 Problem statement

Consider the problem of finding the minimum value of the function [3, 4]



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$f(x, y, z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 1} + \sqrt{z^2 + 1}$ under the linear constraint.

$$x + y + z = 4.$$

$$L_{min} = \sqrt{(x + y + z)^2 + (1 + 1 + 1)^2}.$$

2.2 Geometric synthesis

Each term $\sqrt{a_i^2 + b_i^2}$ can be visualized as the hypotenuse of a right-angled triangle. By aligning three such triangles along the x-axis, the total sum of the hypotenuses is minimized when they form a single straight line AE (Figs. 1, 2 and 3).

2.3 Mathematical verification

While the visual-geometric approach provides an immediate solution, it is rigorously supported by analytical methods. If we define the objective function.

$f(x, y, z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 1} + \sqrt{z^2 + 1}$ subject to $x + y + z = 9$, the use of Lagrange multipliers yields $\frac{x}{\sqrt{x^2 + 1}} = \frac{y}{\sqrt{y^2 + 1}} = \frac{z}{\sqrt{z^2 + 1}} = \lambda$. This symmetry implies

$x = y = z = 3$, which perfectly aligns with our geometric straight-line construction. Integrating this verification into the curriculum allows students to bridge the gap between abstract calculus and physical intuition (Table 1).

2.4 Result

Using the Pythagorean theorem for the combined triangle with base.

$$x + y + z = 4.$$

And total height $1 + 1 + 1 = 3$, we obtain.

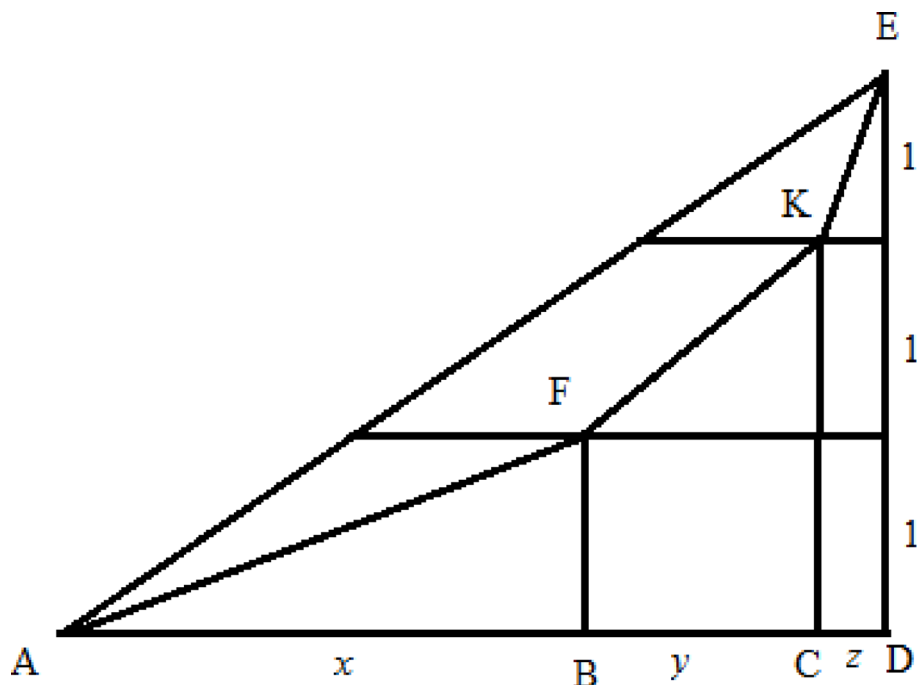


Fig. 1 Geometric visualization of the shortest path for radical sums

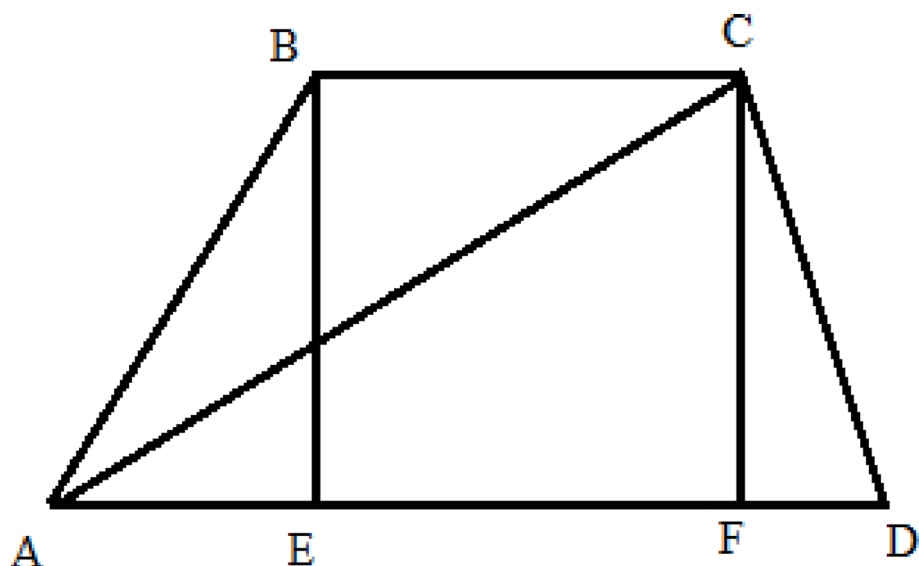


Fig. 2 Largest diagonal

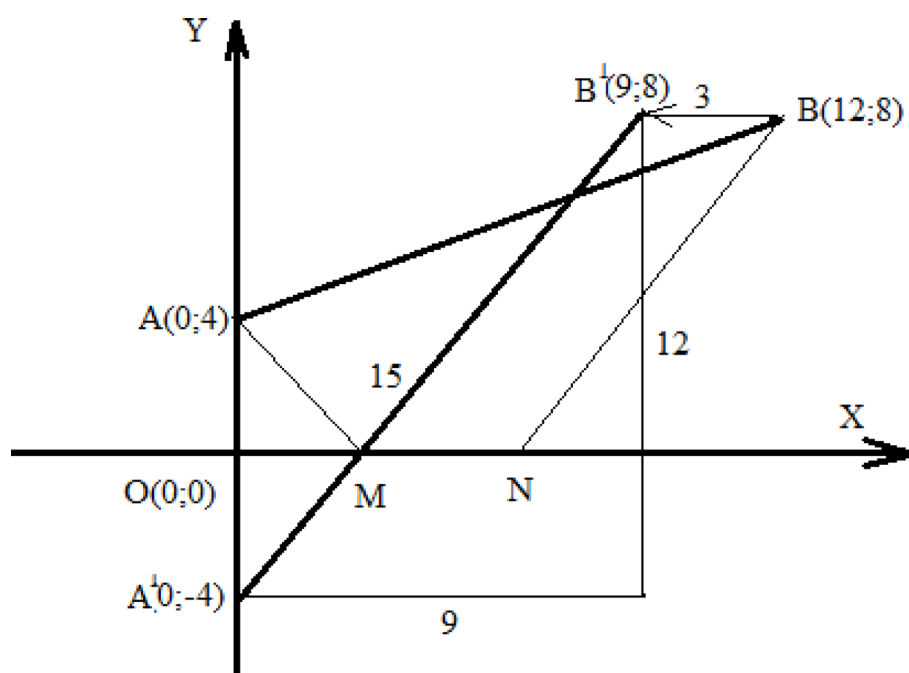


Fig. 3 Geometric construction of the shortest path for AMNB with constant segment MN

$$AE = \sqrt{4^2 + 3^2} = 5.$$

3 Structural design and trapezoidal geometry

3.1 Optimization in structural elements

In mechanical and civil engineering, the optimization of diagonal supports in trapezoidal structures is a common design task. Given a trapezoid with a fixed area $S = 10$, we seek to find the maximum possible length of the shorter diagonal l [5, 6].

Table 1 Comparison of optimization techniques

Problem Case	Calculus Approach (Derivative)	Geometric Approach (Visual)	Efficiency Gain	Complexity Reduction
Radical Sums (Eq. 1)	Complex partial derivatives	Pythagoras theorem & reflection	High (Immediate)	High (~70% reduction in steps)
Trapezoid Diagonals	Second-order optimization	AM-GM Inequality & Geometry	Medium	Significant (~50% faster derivation)"
Transcendental Numbers	Logarithmic differentiation	Functional Monotonicity	High	Conceptual (Immediate identification)
Shortest Path (MN segment)	System of equations	Parallel Translation	Very High	Very High (Eliminates multi-variable calculus)

3.2 Mathematical modeling

The relationship can be expressed as $l = \sqrt{\left(\frac{S}{H}\right)^2 + H^2}$, where H is the height. Applying geometric constraints and the property of arithmetic and geometric means (AM-GM), $\frac{\left(\frac{S}{H}\right)^2 + H^2}{2} \geq \sqrt{\left(\frac{S}{H}\right)^2 H^2} = S, l^2 \geq 2S, l = \sqrt{2 \cdot 10} = 2\sqrt{5}$, the maximum value is found to be $2\sqrt{5}$.

4 Practical applications in transportation electrification

The theoretical optimization methods discussed in the previous sections find direct applications in the field of modern transportation engineering, particularly in the design and management of electric vehicles (EVs).

4.1 Motor efficiency and radical sum optimization

In the analysis of permanent magnet synchronous motors (PMSM), the relationship between current density, magnetic flux, and torque ripple often involves non-linear radical expressions. Traditionally, engineers rely on iterative numerical methods or complex derivatives to find the efficiency peak. By applying the visual-geometric optimization of radical sums, it is possible to graphically represent the power-loss boundaries. This intuitive approach allows for the rapid identification of optimal operating points that minimize copper and iron losses, thereby extending the driving range of the EV.

4.2 Battery pack configuration and trapezoidal geometry

The structural layout of battery modules within an EV chassis is often constrained by non-rectangular spaces, typically resulting in trapezoidal cross-sections. Optimizing the internal area and the diagonal lengths of these sections is crucial for maximizing energy density while ensuring sufficient space for thermal management systems. Our proposed geometric strategy for trapezoids provides a simplified framework for mechanical engineers to calculate optimal cell placement and cooling channel routing without the need for high-level calculus-based modeling.

4.3 Infrastructure planning: optimal cable pathing

A practical engineering problem arises in the deployment of charging infrastructure. Consider a scenario where a central power unit must be connected to two disparate charging docks (Terminals A and B) with the shortest possible cable length. Using the principles of Steiner-point optimization derived from our visual-algebraic methodology, we can determine the "Torricelli point" of the layout. This geometric solution offers an immediate physical intuition for the problem, demonstrating that pedagogical shifts

toward visual modeling can directly improve the efficiency of large-scale infrastructure planning.

5 Comparative analysis of transcendental numbers (e^π vs π^e)

5.1 Problem context

Comparing fundamental constants is a classic optimization task that reinforces the concept of function behavior without direct numerical computation. We investigate the relationship between e^π and π^e .

5.2 Functional modeling

Consider the function $f(x) = \frac{\ln x}{x}$, for $x > 0$. To find the local extremum, we compute the first derivative: $f'(x) = \frac{1 - \ln x}{x^2}$.

Setting $f'(x) = 0$, yields the critical point at $x = e$. Since $f'(x) > 0$ for

$x < e$ and $f'(x) < 0$ for $x > e$, the function reaches its absolute maximum at $x = e$.

5.3 Final proof

Since $e < \pi$, we have $f(e) > f(\pi)$, $\frac{\ln e}{e} > \frac{\ln \pi}{\pi}$, $\pi \ln e > e \ln \pi$, $e^\pi > \pi^e$.

Thus, we conclude that $e^\pi > \pi^e$.

6 Optimization of volumetric efficiency in cylindrical geometry

6.1 Engineering application

Maximizing the volume of a cylindrical container with a fixed surface area is a standard constraint in materials science and packaging engineering [7, 8].

6.2 Mathematical derivation

For a cylinder with a total surface area $S_{total} = 24\pi$, we define the area as.

$2\pi R^2 + 2\pi RH = 24\pi$. Expressing height H in terms of radius R : $H = \frac{12 - R^2}{R}$.

The volume function $V(R)$ becomes:

$$V(R) = \pi R^2 H = 12\pi R - \pi R^3.$$

6.3 Optimization result

Differentiating $V(R)$ with respect to R and setting it to zero:

$$V'(R) = 12\pi - 3\pi R^2, R = 2.$$

Substituting $R = 2$ back into the height equation, we find $H = 2$. This proves that for maximum volume under these constraints, the height must equal the radius.

7 Advanced geometric optimization: the shortest path with a fixed segment

7.1 Problem formulation

In logistical engineering and path planning, finding the shortest route between two points with a fixed-length constraint is a vital task. We consider two points A(0, 4) and B(12, 8) in a Cartesian coordinate system. The objective is to find the minimum length of a path AMNB, where M and N lie on the x-axis, and the segment MN = 3 is a horizontal translation of constant length.

7.2 Methodology: parallel translation and reflection

To determine the optimal position of M and N, we apply a two-step geometric transformation:

1. *Translation*: We shift point B(12, 8) horizontally to the left by the distance $MN = 3$, resulting in a virtual point B'(9, 8).
2. *Reflection*: We reflect point A(0, 4) across the x-axis to A'(0, -4).
3. *Synthesis*: The shortest path between A' and
4. B' is a straight line, representing the hypotenuse of a right-angled triangle.

7.3 Quantitative analysis

The total minimum distance L is calculated using the distance formula between A'(0, -4) and B'(9, 8):

$$L_{min} = \sqrt{(9-0)^2 + (8-(-4))^2} + 3 = \sqrt{9^2 + 12^2} + 3 = 15 + 3 = 18.$$

7.4 Path minimization analysis

The points M(3, 0) and N(6, 0) represent the optimal transition segment along the horizontal infrastructure. By maintaining the rigid constraint $MN = 3$, the total path distance is minimized when the segments AM and NB are parallel to the virtual straight line A'B'. This visual proof reduces the complexity of traditional multi-variable optimization often found in transportation electrification models.

8 Future work and pedagogical extensions

The current study focuses on 2D and 3D Euclidean geometry cases for engineering optimization. Future research could extend these "visual-algebraic" strategies to more complex engineering domains, such as optimization in non-Euclidean spaces and its application in machine learning algorithms. Additionally, empirical studies are planned to measure the impact of these methods on the learning speed of first-year engineering students. This will provide quantitative data to support the qualitative advantages discussed in this paper.

9 Conclusion

This research demonstrates that while differential calculus remains the cornerstone of engineering mathematics, the integration of visual-geometric and multi-methodological approaches significantly enhances pedagogical outcomes. By analyzing complex optimization problems—from radical sums to structural trapezoidal geometry—we have shown that intuitive "short-cut" strategies reduce cognitive load and provide a deeper physical understanding of mathematical modeling. Furthermore, the practical applications discussed in relation to electric vehicle energy efficiency and battery packaging highlight the real-world utility of these methods in modern transportation electrification. Future work will focus on empirical studies to quantify the impact of these visual strategies on the problem-solving speed and analytical accuracy of engineering students in interdisciplinary environments.

Author contributions

A.N. conceived the study, developed the mathematical models and visual-geometric optimization strategies, and drafted the manuscript. All authors reviewed and approved the final manuscript.

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No datasets were generated or analysed during the current study.

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Consent for publication

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Competing interests

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