RESEARCH

Leveraging Linear Programming-based T-spherical Fuzzy AROMAN Method for Vehicle Routing Software Selection in Last-mile Logistics

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Abstract
This paper introduces a novel approach, the linear programming-based T-spherical fuzzy alternative ranking order method accounting for two-step normalization (AROMAN), designed to assist last-mile delivery (LMD) companies in the advanced selection of vehicle routing software (VRS). Selecting an effective VRS is a critical yet formidable challenge for LMD organizations due to the overwhelming variety of available software options and the inherent complexity of urban logistics. These operations involve intricate resource allocation, schedules sensitive to real-time traffic, and the constant need to align delivery paths with rapidly evolving customer preferences. Furthermore, the LMD environment is characterized by significant uncertainties and multi-dimensional constraints that traditional selection methods often fail to capture. By incorporating T-spherical fuzzy logic and linear programming principles, the proposed method effectively models these complexities and uncertainties, providing a methodical framework for assessing VRS alternatives based on a diverse array of criteria. The integrated model is validated through a practical case study of an LMD company operating in England. A panel of experts established specific evaluation criteria to identify the most favorable VRS solution. This research contributes to the field of logistics and transportation management by presenting a robust, data-driven methodology that enables LMD companies to improve their competitiveness and operational efficiency in dynamic marketplaces.

Keywords T-spherical fuzzy AROMAN · Linear programming · Multi-criteria decision making · Vehicle routing software · Last-mile delivery · Two-step normalization · Logistics optimization

1 Introduction

The consumer behavior landscape has seen significant transformations throughout time, mostly driven by societal changes and the emergence of technology. The widespread implementation of e-commerce has been greatly expedited by the lockdown restrictions enforced during the pandemic [1]. The worldwide e-commerce industry experienced a significant surge in 2020, reaching a sales volume of 4.2 trillion US dollars. This represents a remarkable growth rate of 26.76% compared to the previous year. The increase in e-commerce has led to a shift in consumer expectations, with a demand for quick delivery, more frequent ordering, and shorter delivery times.

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This necessitates a wide array of options for home delivery [2]. Despite the high expectations, significant problems, particularly inefficiency, result in delayed or compromised deliveries. This can have a substantial impact on customer happiness, leading to an unsatisfied clientele and a consequent rise in customer complaints, a problem that is prevalent in all industries. Paradoxically, as customers increased their expectations [3], they became more unwilling to pay the necessary price to compensate suppliers for the expenses invested in meeting the demanding logistical requirements. The reluctance to engage in last-mile activities has had a significant impact on urban logistics, highlighting the need for last-mile delivery (LMD) in modern urban systems [4].

Logistics providers frequently encounter difficulties during the LMD process due to the numerous complexities that must be managed, including resource allocation, complex route planning, diverse product assortments, and the provision of services to multiple delivery locations [5]. Another challenge facing an organization is the need to align delivery with customer preferences especially when this factor has been traditionally dictated by the operational efficiency-oriented systems, procedures, and regulations that lack the ability to keep up with changing customer needs. The attempts of the firms to make their products and services more palatable is an essential strategy. This pertains to the development of strategies to reduce expenses associated with petroleum, personnel, planning time, and environmental costs related to CO_2 emissions [6]. A prime example of a minimization problem that has been analyzed in detail is the vehicle routing problem, which is a field of study in the sphere of logistics and transportation. The vehicle routing problem aims at minimizing costs by finding ways of making the routes used by a vehicle fleet most cost-effective so that they can serve a given number of consumers. Such routes usually start and at times end at a hub. The use of algorithms to determine the most efficient and cost-effective paths is essential to the reduction of costs and time savings. This optimizes routing to assist organizations in addressing immediate challenges and lays the foundation for sustained expansion and progress within the fiercely competitive logistics industry [7].

Many organizations are using software packages that purport to handle the complexities of automobile routing problems. Though some researchers outline the basic features that are included into the effective routing software, such as the number of vehicles, speed, and type, not every existing solution is as beneficial, convenient, and easy to access. It is crucial for organizations seeking to optimize their LMD operations to select the most suitable vehicle routing software (VRS) [8]. The choice of a VRS may significantly influence the effectiveness of LMD companies since the success of route optimization leads to better customer satisfaction and reduced costs. The main aim of the research is to offer a proposal to the LMD firms on the appropriate selection process of VRS. Organizations will choose a VRS strategy in accordance with their aims and objectives [9]. Consequently, they will enhance operational efficiency and increase their market competitiveness by developing a more comprehensive comprehension of the operations they are carrying out and the particular demands and specifications associated with them [10].

1.1 Literature Review

When Zadeh [11] introduced the idea of “fuzzy sets” (FS), it significantly improved how ambiguity was handled in decision-making contexts. Regardless of how erroneous or perplexing the data was, Zadeh was able to understand and navigate the maze of decision-making because to his mathematical method. Atanassov [12] enhanced the fuzzy set’s ability to handle complicated decision-making situations by researching membership and non-membership features and introducing the concept of "intuitionistic fuzzy sets" (IFS). This was made possible by using information from earlier studies [13]. Cuong [14] claims that "Picture fuzzy sets" (PFS) are a helpful technique for producing more comprehensive models that accurately represent human viewpoints during decision-making. Cuong and Hai underlined the essential operators and significant features of the system in their research [15, 16]. Deva and Mohanaselvi [17] combined the Choquet integral with PFS and presented geometric aggregation techniques based on the image fuzzy Choquet integral to enhance multi-attribute decision-making. Because the sum of the values was more than one, there seemed to be restrictions on PFSs, which led to the development of "spherical fuzzy sets" (SFS) [18, 19]. Tahir et al. [20] proposed T-SFS as a continuation of PFS and SFS. Parametric similarity measures were proposed by Sarfraz and Azeem [21] for use with T-SFSs.

Bošković et al. [22] suggest AROMAN, a unique ranking order method that makes use of two-step normalization, to improve decision-making accuracy and effectiveness. Selecting electric cars is a nice example of when this strategy works well. MPSI, DNMARCOS, AROMAN, and MACONT are combined by Güclü [23] to assess and compare various MCDM techniques. This comparison and contrast offer fresh perspectives on the effectiveness of different methods in diverse decision-making scenarios. Vasudevan et al. [24] developed and confirmed a computational fluid dynamics modeling technique for both urban and isolated street canyon situations using data from wind tunnels. The interval-valued intuitionistic fuzzy AROMAN approach is offered by Alrasheedi et al. [25] as part of their research on the selection of environmentally suitable wastewater treatment systems. In cases of uncertainty, this method offers a solid foundation for decision-making. In their hybridized fuzzy-AROMAN-Fuller method to professional driver selection, Čubranić-Dobrodolac et al. [26] propose a thorough model that tackles the difficulties of decision-making. Nikolić et al. [27] systematically increase the operational efficiency and reduce the environmental impact of rural postal networks by utilizing an interval type-2 fuzzy AROMAN algorithm. The goal of this strategy is to make these networks more resilient. Pishahang et al. [28] show that AROMAN is useful in handling difficult decision-making situations in disaster management scenarios by using MCDM to assess wildfire risk in Arizona. Their skill in resolving these problems demonstrates this. According to Bošković et al. [29], concepts for cargo bike delivery are selected using the AROMAN approach. They also show that this approach works well in a variety of decision-making situations. Dundar [30] looks at how successful practical entrepreneurial trainings are given in different contexts using fuzzy BWM and AROMAN approaches. His method for doing this is to write reviews of the concerts.

The literature has a wide range of techniques and approaches to decision-making. Martin and Edalatpanah [31] recommend selecting nanomaterials-especially those whose performance measures are influenced by gender-by using the extended fuzzy ISOCOV technique. Image Integral Geometric Aggregation with Fuzzy Choquet Deva and Mohanaselvi [32] suggest using operators in multi-attribute decision-making. Ramana et al. [33] address the issue of establishing priorities for settings that are ambiguous, taking uncertainties into account throughout the process of identifying the best locations for solar power plants. Using an integrated spherical fuzzy SWARA-WASPAS technique, Qiu et al. [34] provide strategies for improving Industry 4.0 deployment in East Africa. The use of integrated approaches is emphasized as a means of handling complicated decision-making circumstances. Berbiche et al. [35] examine, within a fuzzy adaptive framework, the impact of automation on supply chain resilience and efficiency in stormy conditions. The effectiveness of the European Union member states' attempts to mitigate climate change is assessed by Puška et al. [36] using a modified MABAC approach that incorporates fuzzy numbers. Using this approach provides policymakers and decision-makers with more information. In addition, Nanduri et al. [37] propose a modified fuzzy method that might greatly enhance sentiment analysis and natural language processing by automatically classifying cyber-hate speech. More instances of fuzzy logic are found in [38–41].

The research on quality assurance in the food industry illustrates how hybrid fuzzy models successfully translate qualitative expert knowledge into actionable industrial data [42]. Furthermore, the modeling of pedestrian dynamical behavior and driver decision-making highlights the efficacy of fuzzy logic in simulating the inherent uncertainties and complexities of human judgment in motion-based environments [43, 44]. Collectively, these studies underscore the critical role of membership functions and the defuzzification process in converting ambiguous linguistic evaluations into the precise mathematical values required for the T-spherical fuzzy AROMAN method. By incorporating these perspectives, this paper reinforces the link between foundational fuzzy set theory and the rigorous optimization necessary for modern last-mile logistics management.

1.2 Motivation and Contribution

Emerging dynamics of last-mile delivery operations are being altered by the constantly changing customer demands. Thus, it is necessary to develop a VRS system that works effectively. With the constantly growing demands and requirements of operational efficiency, a great number of LMD organizations have to make the hard choice of which VRS would be installed. Nevertheless, the overwhelming number of VRS options and the nature of the LMD business make a significant challenge to companies in search of the perfect solution. It is, therefore, urgent to find a way in which to incorporate advanced mathematical modeling and fuzzy logic within a single platform, which would be beneficial to the LMD firms in the procedure of identifying the appropriate VRS answer.

To choose the VRS, this paper proposes a new method named the T-spherical fuzzy which is a linear-programming-based ranking order method using two-step normalization to select VRS for LMD companies. In this respect, the proposed model offers the researchers a systematic way of how to select the VRS alternatives in the area of logistics and transport management systematically, considering a variety of criteria. The application of this approach was further supported by the fact that the integrated model was applied to a case study in one of the LMD companies in England. The methodology is capable of dealing with the ambiguity and complexity that is a feature of LMD operations by utilizing T-spherical fuzzy logic and linear-programming principles. The ability can make organizations find the best solutions of VRS. The results shown in this paper appear to support the indices obtained through the case study of an LMD company located in England. This study augments viable and sound methodologies of making VRS choice, thus helping LMD firms in dynamic marketplaces to gain competitiveness and operational effectiveness.

1.3 Structure of the Paper

This paper is structured as follows: To provide the groundwork for the suggested method, Section 2 examines the basic ideas of T-SFS theory. The suggested work's technique, namely the T-spherical fuzzy AROMAN method based on linear programming, is described in Section 3. Section 4 delves into the pragmatic applications of the suggested methodology, furnishing insightful insights on potential approaches for executing the plan to tackle the problem. In Section 5, the research's practical consequences are examined, theoretical limitations are examined, and empirical results are summarized. The topic is brought to a close in Section 6 with recommendations for further reading.

2 Preliminaries

Definition 1 [20] A T-SFS in U is defined as:

$$\psi = \{\langle v, M(v), N(v), L(v) | h \in U \rangle\}, \tag{1}$$

where $M(v), N(v), L(v) \in [0, 1]$, such that $0 \leq M(v) + N(v) + L(v) \leq 1$ for each $v \in U$. $M(v), N(v), \text{and} L(v)$ represent membership degree (MD), abstention degree (AD), and non-membership degree (N-MD) for some $v \in U$ .This pair is represented by the expression $D = (M_v, N_v, L_v)$. It is called T-SFN throughout this article. It must meet the following requirements: $M_v + N_v + L_v \leq 1$; $M_v, N_v, L_v \in [0, 1]$.

Definition 2 [20] Prior to being used in real-world scenarios, T-spherical fuzzy numbers (T-SFNs) need to be categorized. T-SFN is the appropriate "score function" (SF) in this case. This is how we will define *SF*:

$$SF(Y) = M_Y - L_Y. \tag{2}$$

It is challenging to decide which is superior, nonetheless, since the previously described function is insufficient for classifying T-SFNs in diverse circumstances. Finding an accuracy function Q of L achieved, for example, in the method that follows:

$$Q(Y) = M_Y + N_Y + L_Y. \tag{3}$$

We will provide guidelines for the operational aggregation of T-SFNs.

Definition 3 [45] Let $\mu_1 = \langle M_1, N_1, L_1 \rangle$ and $\mu_2 = \langle M_2, N_2, L_2 \rangle$ be two T-SFNs, then:

$$\mu_1^c = \left\langle L_1, N_1, M_1 \right\rangle, \tag{4}$$

$$\mu_1 \vee \mu_2 = \left\langle max\{M_1, M_2\}, min\{N_1, N_2\}, min\{L_1, L_2\} \right\rangle, \tag{5}$$

$$\mu_1 \wedge \mu_2 = \left\langle min\{M_1, M_2\}, max\{N_1, N_2\}, max\{L_1, L_2\} \right\rangle, \tag{6}$$

$$\mu_1 \oplus \mu_2 = \left\langle \sqrt[t]{M_1 + M_2 - M_1 M_2}, N_1 N_2, L_1 L_2 \right\rangle, \tag{7}$$

$$\mu_1 \otimes \mu_2 = \left\langle M_1 M_2, \sqrt[t]{N_1 + N_2 - N_1 N_2}, \sqrt[t]{L_1 + L_2 - L_1 L_2} \right\rangle, \tag{8}$$

$$\sigma \mu_1 = \left\langle \sqrt[t]{1 - (1 - M_1)^\sigma}, N_1^\sigma, L_1^\sigma \right\rangle, \tag{9}$$

$$\mu^\sigma = \left\langle M_1^\sigma, \sqrt[t]{1 - (1 - N_1)^\sigma}, \sqrt[t]{1 - (1 - L_1)^\sigma} \right\rangle. \tag{10}$$

Definition 4 Let $\mu_1 = \langle M_1, N_1, L_1 \rangle$ and $\mu_2 = \langle M_2, N_2, L_2 \rangle$ be two T-SFNs and $\mathbb{A}, Al_1, Al_2 > 0$ be the real numbers; then we have the following:

1. $\mu_1 \oplus \mu_2 = \mu_2 \oplus \mu_1.$
2. $\mu_1 \otimes \mu_2 = \mu_2 \otimes \mu_1.$
3. $\mathbb{A}(\mu_1 \oplus \mu_2) = (\mathbb{A}\mu_1) \oplus (\mathbb{A}\mu_2).$
4. $(\mu_1 \otimes \mu_2)^{\mathbb{A}} = \mu_1^{\mathbb{A}} \otimes \mu_2^{\mathbb{A}}.$
5. $(Al_1 + Al_2)\mu_1 = (Al_1 \mu_1) \oplus (Al_2 \mu_2).$
6. $\mu_1^{Al_1 + Al_2} = \mu_1^{Al_1} \otimes \mu_2^{Al_2}.$

Definition 5 [20] For T-SFNs $S_j = (j = 1, 2, 3, \ldots, s)$, the T-spherical fuzzy weighted geometric (T-SFWG) operator is as follows:

$$\text{T-SFWG}(S_1, S_2, \ldots, S_s) = \prod_{j=1}^{s} S_j^{O_j},$$

where the weighted vector of $O_j = (j = 1, 2, 3, \ldots, s)$, $O_j > 0$.

Theorem 1 *[20] Using the T-SFWG operator, the aggregated value of a set of $T-SFNs$ S_j is likewise a $T-SFN$.*

$$\text{T-SFWG}(S_1, S_2, \ldots, S_s) = \left(\prod_{j=1}^{s} (M_j + N_j)^{O_j} - \prod_{j=1}^{s} N_j^{O_j}, \prod_{j=1}^{s} N_j^{O_j}, \sqrt[t]{1 - \prod_{j=1}^{s} \left(1 - L_j^t\right)^{O_j}} \right). \tag{11}$$

3 Linear Programming-based T-spherical Fuzzy AROMAN Method

Assume that $A = \{Al_1, \ldots, Al_i, \ldots, Al_m\}$ is a collection of *m* options. The notation $C = \{\text{Cr}_1, \ldots, \text{Cr}_j, \ldots, \text{Cr}_n\}$ represents a finite collection of criteria. As an example, let $J = \{J_1, \ldots, J_e, \ldots, J_z\}$ represent the group of DMs invited. The T-spherical fuzzy AROMAN approach based on linear programming is elucidated by the below phases.

Algorithm

Step 1: Determine the weights of "linguistic terms" (LTs) used to represent DMs as T-SFNs. Table 1 exhibits a list of LTs. Given that $\Upsilon_k = M_k, N_k, L_k \langle BigFor\text{k}{-}thDM, Big\rangle$ is the T-SFN. Thus, ζ_k, the value of the *k*-th DM, found as follows:

$$Al_k = \frac{\Upsilon_k}{\sum_{k=1}^{z} \Upsilon_k}, k = 1, 2, 3, \ldots, z \tag{12}$$

where $\Upsilon_k = M_k + (1 - M_k - N_k - L_k) \left(\frac{L_k}{M_k + N_k + L_k} \right)$ and Clearly $\sum_{k=1}^{p} Al_k = 1$

Step 2: The decision matrices ought to be sent to you by the DMs. Table 2 lists the eight linguistic terms that characterize each choice. The large range of linguistic languages that are available makes it possible to depict the information-assessment process in detail.

Step 3: Put the corresponding T-SFN instead of LTs.

Step 4: Remember that all individual opinions need to be integrated to form a collective opinion when using a group decision-making strategy to construct the consolidated T-SF decision matrix. To get the aggregated decision matrix, use T-spherical fuzzy weighted geometric AOs.

Step 5: Compile the scoring matrix by incorporating the SF of T-SFNs as $\Psi = \left(\aleph_{ij}\right)_{m \times n}$.

Table 1 Linguistic terms for DMs

Linguistic terms	T-SFNs
Highly suitable	(0.950, 0.050, 0.050)
Appropriate	(0.900, 0.100, 0.150)
Moderately suitable	(0.850, 0.300, 0.250)
Unappropriate	(0.600, 0.400, 0.500)

Table 2 Linguistic terms for evaluation

Linguistic term	T-SFN
Excellent (VH)	$(\langle 0.95, 0.05, 0.05\rangle)$
Good (Hg)	$(\langle 0.90, 0.10, 0.10\rangle)$
Fair (Md)	$(\langle 0.85, 0.15, 0.15\rangle)$
Satisfactory (Ad)	$(\langle 0.80, 0.20, 0.20\rangle)$
Acceptable (Ac)	$(\langle 0.75, 0.25, 0.25\rangle)$
Marginal (Li)	$(\langle 0.70, 0.30, 0.30\rangle)$
Poor (P)	$(\langle 0.60, 0.35, 0.35\rangle)$

Step 6: According to the concept of a scoring matrix Ψ, the weighted sum of the scores of all alternatives $\mathscr{H}^{\daleth}{}_j$ is determined by

$$¥(\mathscr{H}^{\daleth}{}_j) = \sum_{i=1}^{m} \mathfrak{W}_i^{\gamma} \left(\aleph_{ij}\right).$$

where, $\mathfrak{W}_1^{\gamma}, \mathfrak{W}_2^{\gamma}, \ldots \mathfrak{W}_m^{\gamma}$ be the WV of the given criterion.

Assume that $\widehat{\coprod}$ represents a subset of the weights, which are assumed to be indeterminate. The following mathematical framework is used to compute these unknown weights: $Max\ \ g = \sum_{i=1}^{m} ¥(\mathscr{H}^{\daleth}{}_j)$ under the constraints $\sum_{i=1}^{m} \mathfrak{W}_i^{\gamma} = 1$.We may use this model to calculate our normalized WV. To calculate the weights of the criterion in this instance inside the constraints of the circumstance, we use a linear programming paradigm.

Step 7: Use normalization to ensure that all of the input data in the decision-making matrix is uniform. The data must then be arranged into intervals of 0 to 1 after the input data matrix has been created. Two distinct techniques are used to standardize the data. Equation 13 and 14

Step 7.1: Normalization 1 (**Linear**):

$$Y_{ij} = \frac{\aleph_{ij} - \aleph_{ij}}{\aleph_{ij} - \aleph_{ij}}, \quad i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n; \tag{13}$$

Step 7.2: Normalization 2 (**Vector**):

$$Y_{ij}^{*} = \frac{\aleph_{ij}}{\sqrt{\sum_{i=1}^{m} \aleph_{ij}^{2}}}, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, s; \tag{14}$$

Step 8: Use averaged aggregation normalization to standardize the supplied data. This systematic methodology ensures consistent and meaningful comparisons across a variety of criteria throughout the data normalization process. The aggregated averaged normalization process use Equation 15.

$$Y_{ij}^{\text{norm}} = \frac{\chi Y_{ij} + (1-\chi) Y_{ij}^{*}}{2}, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, s; \tag{15}$$

where Y_{ij}^{norm} is the aggregated average normalization result, and χ is a weighting factor in the range of 0 to 1. To account for the specific setting we are in, we set χ to 0.5.

Step 9: To generate a weighted decision-making matrix, multiply the aggregated averaged normalized decision-making matrix by the criterion weights, as per Equation 16.

$$\hat{Y_{ij}} = \mathfrak{W}_{ij}^{\gamma} \cdot Y_{ij}^{\text{norm}}, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, s \tag{16}$$

Step 10: Give the normalized weighted values for the criterion types $\min(x_i)$ and (x_i) in explicit terms using Equation 17. This formula ensures a comprehensive representation of the needs under minimization and maximization situations by providing a clear foundation for computing the weighted normalized values.

$$\begin{aligned} x_{T(i)} &= \sum_{j=1}^{n} \widehat{Y_{ij}}^{(\min)}, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, s; \\ s_{T(i)} &= \sum_{j=1}^{n} \widehat{Y_{ij}}^{(\max)}, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, s; \end{aligned} \tag{17}$$

Step 11: Determine the ultimate order of the alternatives by considering all relevant data and evaluations. This final rating provides a clear order that reflects the alternatives' overall performance and suitability for use in decision-making. It is based on the methodology and criteria that were used to thoroughly evaluate the options.

$$R_i = {x_{T(i)}}^{Z^\lambda} + {s_{T(i)}}^{(1-Z^\lambda)} \tag{18}$$

The ranked alternatives in this case are labeled (R_i), and Z^λ represent the coefficient degree corresponding to the criteria type. Upon consideration of both categories of criteria, Z^λ is assigned a value of 0.5.

The systematic 11-step framework of the linear programming-based T-spherical fuzzy AROMAN method is visually summarized in Figure 1.

4 Statement of the Problem

In the ever-changing logistics and manufacturing distribution environment, the effectiveness of route planning becomes one of the determinants for the companies that want to stay in the competitive advantage and be operationally excellent. The paper is a critical examination of the complex issue that an LMD organization encounters when deciding on an efficient VRS. Considering the intricacy of its supply chain and the evolving needs of its customers, the company needs to find a VRS that smoothly fits its operation complexities and its priorities [46]. In this context, the current question explores the subtleties of the selection procedure, with a focus on such aspects of the matter as optimization of routes, cost-effectiveness, flexibility, and scalability. Explaining the course of VRS choice, the case study brings into the limelight the strategic choices and the transformational implications for the LMD firm in terms of logistical efficiency and service reliability. This investigation aims at providing a feasible solution to VRS selection dilemma that an LMD company based in England is faced with.

4.1 Definition of Alternatives

- **RouteSmart** Al_1: RouteSmart is a well-known organization worldwide due to their contribution to optimization in the logistics operation. Its software package provides rerouting solutions that are industry-specific and new that solve challenges in the industry. Since RouteSmart was launched in 1992, it has been a leader in revolutionizing methodology used in route planning by taking advantage of advanced technologies to model complex situations in a bid to increase efficiency and decrease operations costs. RouteSmart is also renowned with its firm adherence to the success of its clients in addition to offering the complete consulting and implementation that streamlines the integration process and maximizes the value provided to the customer. The organization itself is a representation of progressive resourcefulness and has the experienced knowledge that will help clients to optimize the logistics process and maintain a competitive advantage in the contemporary market [47].

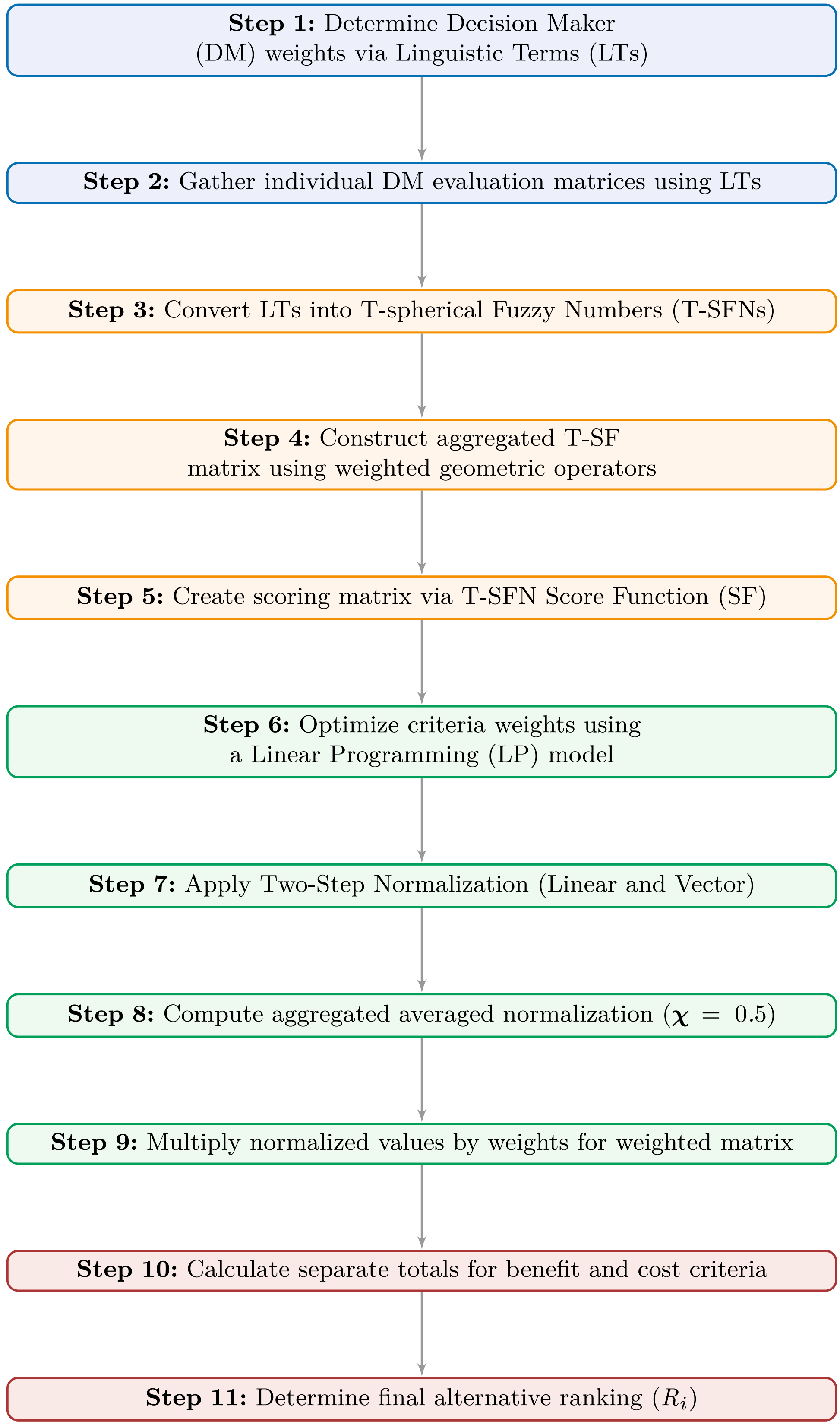

Fig. 1 Systematic S11-step framework of the proposed methodology

- **SalesArt** Al_2: SalesArt is a customized software platform that has been specifically designed to address the complex issues of LMD entities in the process of key decision-making. The platform is compatible with the existing logistical architectures, whereby it utilizes advanced algorithms and user-friendly interfaces. LMD organizations use SalesArt to calculate various paths based on different routing options, considering the various parameters of the vehicle capacity, delivery time, and geographical limitations in the given service areas. Besides, the solution will provide a set of customized features that meet unique firm-specific needs, which will guarantee the most effective allocation and use of resources. Implementing SalesArt will enable LMD enterprises to improve the level of operational efficiency, reduce the costs of transportation and increase the customer satisfaction rates, becoming leaders in the competitive environment [48].
- **Optiyol** Al_3: Optiyol is a pioneer in the field of VRS which is adapted to the LMD settings that have the task of facing the intricate routing challenges. The platform is developed specifically to meet the demand of complex logistics and distribution with a clean set of advanced algorithms that are presented in a client-friendly format to the end user. Optiyol is the company that is focused on constant improvement and client needs. Optiyol can be incorporated into the systems of clients, which will allow maximum routing, cost savings, and increased efficiency. Using its complete functionality and high functionality, Optiyol eases the smooth sailing of LMD enterprises through the constantly changing environment of transportation logistics. As the agent of change, it provides dynamic and versatile VRS solutions, fulfilling the increasing market needs and leading organizations on their way to greater success [49].
- **Geovision** Al_4: It is interesting to note the effectiveness of the Geovision application in solving a VRS selection problem of an LMD firm. Geovision provides a complex routing system that integrates complex optimization algorithms with geographic maps. The powerful functionality and user-friendly interface of the platform allow the firms to address most of the obstacles related to the route planning and achieve substantial operational and financial gains. Its dynamic capabilities variable its route allocations in real-time conditions by responding to changing constraints, scheduling of deliveries, and vehicle capacities. Along with optimization of routing, Geovision also offers data-intensive decision support, following which interdependent logistical processes are optimized and, at the end, the level of customer satisfaction is increased. By using the Geovision services in a strategic manner, companies will be in a position to operate in the complicated distribution logistics environment with accuracy and achieve long-term growth in their businesses and sustainable competitive advantage [50].
- **Logvrp** Al_5: Logvrp is one of the alternatives that will be pivotal to LMD companies intending to avoid the VRS selection challenges. The tool is concerned with the optimization of complex routing processes, and hence it is a pragmatic tool that considers various constraints and operational complexities. Its advanced algorithms are offered through convenient management related to the fleet and mastering the delivery process. Organizations gain a significant amount of resource allocation, cost reduction, and service quality by using the adaptive qualities of Logvrp. The ability of the system to adapt to changing business needs and dynamic business environments is invaluable to organizations that aim at surviving in the highly competitive contemporary logistics industries. Accordingly, the dilemma of VRS selection is redefined as a chance of sustainable growth and operational excellence with the help of the Logvrp [51].
- **SELCO** Al_6: As a supplier of new logistics solutions, SELCO provides a personalized route-optimal solution that is finely adjusted to the peculiarities of the industry. Through the application of advanced algorithms and customized techniques, SELCO provides holistic results to respond to inherent multivariate limitations of LMD activities. This also makes it easy to incorporate the best routing solutions within the logistical system of the organization and, consequently, improve operational efficiency, cost reduction and exploiting the available resources by joining forces and conducting thorough research. The resilience of SELCO towards constant improvement means that the users will acquire the most recent innovations, thus making its VRS solutions an industry leader and capable of mastering the complexities of modern logistics with a lot of confidence and accuracy [52].

4.2 Definition of Criteria

- **Accessibility** Cr_1: In the context of LMD, accessibility is a paramount consideration for ensuring that routes can effectively navigate dense urban road infrastructures and reach localized delivery points within minimal travel time [2]. This criterion evaluates the software's ability to account for one-way streets, vehicle size restrictions in residential zones, and real-time traffic congestion to optimize the final approach to the customer's doorstep [53].
- **Security** Cr_2: It guarantees the safety of goods during shipment, helps to prevent theft or manipulation of goods during transit, and guarantees the safety of drivers and vehicles during the routing process [54]. The variables to be evaluated when comparing the security features of different options include sight-line of the routes and real-time functionality as well as functionality in integrating with other security systems [55]. The choice of the VRS system as a security system is of great importance, which is the security of assets, operation integrity, and reduction of risks in logistics and distribution operations have been addressed through the system. The security of the system is the selection of the VRS system, which protects the assets, operation integrity and minimizes the risks of logistics and distribution operations [7].
- **Total cost of ownership** Cr_3: The total cost of ownership (TCO) consists of the summation of initial VRS purchase prices and the cost of future maintenance, fuel, labor, and possible outages. The main aim is to use TCO as a key factor when making sound decisions that will ensure proper deployment of the best possible routing solutions, thus lowering operation costs in the long term without necessarily implementing untested communication solutions [56]. It also ensures that the implemented VRS would comply with the financial goals of the company and help to develop the sustainable and appropriate cost-management strategies within the logistics and distribution industry [5].
- **Complexity of software** Cr_4: This dimension evaluates the VRS solution in terms of its flexibility, ease of use and surrounding that is compatible with the current technical environment of the firm. The implementation of a less complicated software solution can be done in a more efficient manner, which will lead to less time spent on training and a risk of disruptions in the operation is also lower. Through the analysis of software complexity, it will be possible to check that the chosen VRS does not contradict the technological environment and functional standards of the organization, which improves the efficiency of the routing and distribution activities [9].
- **Functionality** Cr_5: The ability of the system to cope with complex routing situations, flexibility to respond to new operational needs and its ability to integrate smoothly with the existing logistical infrastructure are included therein [57]. A good VRS must have a wide range of functionalities, including the ability to monitor in real time and route-optimization algorithmic functions, as well as user interfaces that are user-intuitive and user-understandable. To guarantee that the selected system addresses the operational requirements in the organization and promotes efficiency and performance within transport and logistics, it is important to consider the functionality of different virtual reality security systems.
- **Reliability** Cr_6: This includes the consistency of the chosen VRS to offer efficient routes and no interruptions or delays. It entails the ability of the system to respond to the fluctuating circumstances, like heavy traffic or sudden road blockages, and continue to achieve the best schedules on the routes. The fact that the VRS can adequately cope with the fluctuations in demand and provide all the items to the clients on time is part of its reliability. An effective VRS reduces the chances of interruption of the services, thus improving the overall effectiveness and customer satisfaction in the LMD organization [1, 4].

4.3 Determination of Experts

A group of specialists, including those in charge of planning and execution, is assigned to decide which software is best in this particular situation. Making sure an expert panel with a variety of viewpoints was chosen was the main goal of the real-world case study on VRS selection. As a result, it was made up of both field employees,

Table 3 DMs for the evaluation

DM	Experience	Profession	Importance
J_1	26 years	LMD company senior executive-2	Appropriate
J_2	19 years	LMD company field staff	Very appropriate
J_3	10 years	LMD company senior executive-5	Appropriate

Table 4 DMs for the evaluation

DM	Importance	T-SFN	Weights
J_1	Appropriate	0.800, 0.150,0.200	0.310
J_2	Very appropriate	0.900, 0.050,0.100	0.380
J_3	Appropriate	0.800, 0.150,0.200	0.310

Table 5 Evaluations of each alternative

DMs	Alternative	Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6
J_1	Al_1	VH	Ad	Li	Ac	P	Md
	Al_2	Ad	Li	P	VH	Ac	Md
	Al_3	Li	Hg	VH	Md	P	Ac
	Al_4	P	Hg	Md	Ad	Li	VH
	Al_5	Md	Ad	Hg	P	VH	Ac
	Al_6	Md	Hg	Li	Hg	VH	P
J_2	Al_1	Hg	Li	Md	VH	Hg	Ad
	Al_2	Md	VH	Ad	Hg	P	Li
	Al_3	P	Hg	Ac	Ad	Md	VH
	Al_4	Li	Hg	P	Ac	VH	Md
	Al_5	Ac	Ad	Hg	Md	P	Li
	Al_6	Md	Ad	Hg	P	VH	Ac
J_3	Al_1	VH	Ac	Hg	Md	P	Ad
	Al_2	Hg	P	Md	Ad	Li	Ac
	Al_3	Ad	Hg	VH	P	Ac	Li
	Al_4	Li	Md	Hg	VH	Ad	P
	Al_5	Ac	Hg	Li	Md	VH	Ad
	Al_6	Hg	Ad	Hg	P	Hg	Ac

who are in charge of carrying it out, and high-ranking decision-makers, including senior executives. As shown in Table 3, the prior expert group's knowledge and experience levels were acquired via talks and consultations with an LDM business that operates in England. After conferring with every expert in the group, the VRSs were assessed according to predetermined standards.

5 Experimental Results

The T-spherical fuzzy AROMAN process is based on linear programming and consists of the following steps:

Step 1: Using LTs, determine the weights of the DMs. Table 3 provides the DM specification. Table 4 contains the DM weights.

Step 2: For the alternatives listed in Table 2, use LTs to get the decision matrices from the DMs. Table 5 contains the decision matrices obtained from the DMs.

Step 3: Instead of using LTs, add the matching T-SFN, which is provided in Table 6.

Step 4: $M = [M_{ij}]_{r\times s}$ is the ADM. Table 7 presents the findings.

Step 5: Integrate the SF of T-SFNs to create the score matrix.

Table 6 Assessment matrices in the form of T-SFNs

DMs	Alternative	Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6
J_1	Al_1	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.80, 0.15, 0.20\rangle$
	Al_2	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.80, 0.15, 0.20\rangle$
	Al_3	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.65, 0.25, 0.30\rangle$
	Al_4	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.90, 0.05, 0.10\rangle$
	Al_5	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.65, 0.25, 0.30\rangle$
	Al_6	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.50, 0.35, 0.40\rangle$
J_2	Al_1	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.75, 0.20, 0.25\rangle$
	Al_2	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.60, 0.30, 0.35\rangle$
	Al_3	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.90, 0.05, 0.10\rangle$
	Al_4	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.80, 0.15, 0.20\rangle$
	Al_5	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.60, 0.30, 0.35\rangle$
	Al_6	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.65, 0.25, 0.30\rangle$
J_3	Al_1	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.75, 0.20, 0.25\rangle$
	Al_2	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.65, 0.25, 0.30\rangle$
	Al_3	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.60, 0.30, 0.35\rangle$
	Al_4	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.50, 0.35, 0.40\rangle$
	Al_5	$\langle 0.65, 0.25, 0.30\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.60, 0.30, 0.35\rangle$	$\langle 0.80, 0.15, 0.20\rangle$	$\langle 0.90, 0.05, 0.10\rangle$	$\langle 0.75, 0.20, 0.25\rangle$
	Al_6	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.75, 0.20, 0.25\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.50, 0.35, 0.40\rangle$	$\langle 0.85, 0.10, 0.15\rangle$	$\langle 0.65, 0.25, 0.30\rangle$

Table 7 Aggregated decision matrix

Al_i	Cr_1	Cr_2	Cr_3	Cr_4	Cr_5	Cr_6
Al_1	$\langle 0.83, 0.17, 0.12\rangle$	$\langle 0.84, 0.13, 0.28\rangle$	$\langle 0.83, 0.13, 0.25\rangle$	$\langle 0.82, 0.12, 0.21\rangle$	$\langle 0.85, 0.18, 0.23\rangle$	$\langle 0.72, 0.34, 0.11\rangle$
Al_2	$\langle 0.75, 0.23, 0.32\rangle$	$\langle 0.74, 0.26, 0.32\rangle$	$\langle 0.68, 0.21, 0.31\rangle$	$\langle 0.85, 0.23, 0.31\rangle$	$\langle 0.71, 0.26, 0.25\rangle$	$\langle 0.45, 0.22, 0.16\rangle$
Al_3	$\langle 0.85, 0.12, 0.18\rangle$	$\langle 0.71, 0.13, 0.24\rangle$	$\langle 0.76, 0.25, 0.22\rangle$	$\langle 0.76, 0.23, 0.26\rangle$	$\langle 0.75, 0.23, 0.27\rangle$	$\langle 0.73, 0.15, 0.52\rangle$
Al_4	$\langle 0.59, 0.42, 0.47\rangle$	$\langle 0.65, 0.49, 0.45\rangle$	$\langle 0.61, 0.48, 0.54\rangle$	$\langle 0.52, 0.0.45, 0.49\rangle$	$\langle 0.52, 0.44, 0.51\rangle$	$\langle 0.73, 0.42, 0.11\rangle$
Al_5	$\langle 0.63, 0.21, 0.47\rangle$	$\langle 0.73, 0.34, 0.39\rangle$	$\langle 0.67, 0.31, 0.49\rangle$	$\langle 0.74, 0.37, 0.44\rangle$	$\langle 0.63, 0.37, 0.47\rangle$	$\langle 0.82, 0.12, 0.21\rangle$
Al_6	$\langle 0.54, 0.55, 0.61\rangle$	$\langle 0.59, 0.51, 0.67\rangle$	$\langle 0.54, 0.53, 0.63\rangle$	$\langle 0.51, 0.53, 0.51\rangle$	$\langle 0.66, 0.52, 0.52\rangle$	$\langle 0.56, 0.24, 0.21\rangle$

$$M = \begin{array}{c} \\ Al_1 \\ Al_2 \\ Al_3 \\ Al_4 \\ Al_5 \\ Al_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{Cr}_1 & \mathrm{Cr}_2 & \mathrm{Cr}_3 & \mathrm{Cr}_4 & \mathrm{Cr}_5 & \mathrm{Cr}_6 \end{array} \\ \begin{pmatrix} 0.650 & 0.546 & 0.524 & 0.375 & 0.252 & 0.551 \\ 0.582 & 0.425 & 0.335 & 0.232 & 0.351 & 0.532 \\ 0.562 & 0.395 & 0.425 & 0.428 & 0.324 & 0.425 \\ 0.531 & 0.597 & 0.473 & 0.570 & 0.452 & 0.642 \\ 0.457 & 0.576 & 0.481 & 0.446 & 0.462 & 0.526 \\ 0.431 & 0.533 & 0.442 & 0.396 & 0.432 & 0.524 \end{pmatrix} \end{array}$$

Step 6: Suppose that the attribute weights are provided by the DMs with the following partial weight details: $\Psi = \quad 0 \leq \mathfrak{W}_1^\gamma \leq 0.50, \quad 0.10 \leq \mathfrak{W}_2^\gamma \leq 0.55, \quad 0.10 \leq \mathfrak{W}_3^\gamma \leq 0.30, \quad 0.10 \leq \mathfrak{W}_4^\gamma \leq 0.50, \quad 0 \leq \mathfrak{W}_5^\gamma \leq 0.40, \quad 0 \leq \mathfrak{W}_6^\gamma \leq 0.60.$

This data is used to construct the optimization framework that follows:

Table 8 Normalizing a fuzzy decision matrix linearly

1.0000	0.7475	1.0000	0.4231	0.0000	0.5806
0.6895	0.1485	0.0000	0.0000	0.4714	0.4931
0.5982	0.0000	0.4762	0.5799	0.3429	0.0000
0.4566	1.0000	0.7302	1.0000	0.9524	1.0000
0.1187	0.8960	0.7725	0.6331	1.0000	0.4654
0.0000	0.6832	0.5661	0.4852	0.8571	0.4562

Table 9 Fuzzy decision matrix vector normalization

0.4908	0.4307	0.4748	0.3645	0.2662	0.4188
0.4395	0.3353	0.3036	0.2255	0.3708	0.4044
0.4244	0.3116	0.3851	0.4161	0.3423	0.3231
0.4010	0.4709	0.4286	0.5541	0.4775	0.4880
0.3451	0.4544	0.4359	0.4335	0.4881	0.3998
0.3255	0.4204	0.4005	0.3849	0.4564	0.3983

Table 10 Aggregated averaged normalization values

0.3727	0.2946	0.3687	0.1969	0.0666	0.2499
0.2822	0.1209	0.0759	0.0564	0.2106	0.2244
0.2556	0.0779	0.2153	0.2490	0.1713	0.0808
0.2144	0.3677	0.2897	0.3885	0.3575	0.3720
0.1160	0.3376	0.3021	0.2667	0.3720	0.2163
0.0814	0.2759	0.2417	0.2175	0.3284	0.2136

Table 11 Aggregated averaged normalization values

0.0335	0.0324	0.0737	0.0492	0.0113	0.0450
0.0254	0.0133	0.0152	0.0141	0.0358	0.0404
0.0230	0.0086	0.0431	0.0622	0.0291	0.0145
0.0193	0.0405	0.0579	0.0971	0.0608	0.0670
0.0104	0.0371	0.0604	0.0667	0.0632	0.0389
0.0073	0.0303	0.0483	0.0544	0.0558	0.0385

$$
\begin{aligned}
Max\ \ g =&0.650\mathfrak{W}_1^\gamma + 0.582\mathfrak{W}_1^\gamma + 0.562\mathfrak{W}_1^\gamma + 0.531\mathfrak{W}_1^\gamma + 0.457\mathfrak{W}_1^\gamma + 0.431\mathfrak{W}_1^\gamma \\
&0.546\mathfrak{W}_2^\gamma + 0.425\mathfrak{W}_2^\gamma + 0.395\mathfrak{W}_2^\gamma + 0.597\mathfrak{W}_2^\gamma + 0.576\mathfrak{W}_2^\gamma + 0.533\mathfrak{W}_2^\gamma \\
&0.524\mathfrak{W}_3^\gamma + 0.335\mathfrak{W}_3^\gamma + 0.425\mathfrak{W}_3^\gamma + 0.473\mathfrak{W}_3^\gamma + 0.481\mathfrak{W}_3^\gamma + 0.442\mathfrak{W}_3^\gamma \\
&0.375\mathfrak{W}_4^\gamma + 0.232\mathfrak{W}_4^\gamma + 0.428\mathfrak{W}_4^\gamma + 0.570\mathfrak{W}_4^\gamma + 0.446\mathfrak{W}_4^\gamma + 0.396\mathfrak{W}_4^\gamma \\
&0.252\mathfrak{W}_5^\gamma + 0.351\mathfrak{W}_5^\gamma + 0.324\mathfrak{W}_5^\gamma + 0.452\mathfrak{W}_5^\gamma + 0.462\mathfrak{W}_5^\gamma + 0.432\mathfrak{W}_5^\gamma \\
&0.551\mathfrak{W}_6^\gamma + 0.532\mathfrak{W}_6^\gamma + 0.425\mathfrak{W}_6^\gamma + 0.642\mathfrak{W}_6^\gamma + 0.526\mathfrak{W}_6^\gamma + 0.524\mathfrak{W}_6^\gamma
\end{aligned} \tag{19}
$$

such that,

$0 \le \mathfrak{W}_1^\gamma \le 0.50, 0.10 \le \mathfrak{W}_2^\gamma \le 0.55, 0.10 \le \mathfrak{W}_3^\gamma \le 0.30, 0.10 \le \mathfrak{W}_4^\gamma \le 0.50, 0 \le \mathfrak{W}_5^\gamma \le 0.40,$
$0 \le \mathfrak{W}_6^\gamma \le 0.60, \mathfrak{W}_1^\gamma + \mathfrak{W}_2^\gamma + \mathfrak{W}_3^\gamma + \mathfrak{W}_4^\gamma + \mathfrak{W}_5^\gamma + \mathfrak{W}_6^\gamma = 1, \mathfrak{W}_1^\gamma, \mathfrak{W}_2^\gamma, \mathfrak{W}_3^\gamma, \mathfrak{W}_4^\gamma, \mathfrak{W}_5^\gamma, \mathfrak{W}_6^\gamma \ge 0.$

By solving this model we get, $\mathfrak{W}_1^\gamma = 0.09, \mathfrak{W}_2^\gamma = 0.11, \mathfrak{W}_3^\gamma = 0.20, \mathfrak{W}_4^\gamma = 0.25, \mathfrak{W}_5^\gamma = 0.17, \mathfrak{W}_6^\gamma = 0.18.$

Step 7: The fuzzy decision matrix is normalized using equations 13 and 14. Tables 8 and Table 9 include the linear form normalization provided by Equations 13 and the vector form normalization provided by Equations 14, respectively.

Step 8: Calculate the aggregated averaging normalization values by using Table 10's χ value of 0.50.

Step 9: Compute the weighted decision matrix, given in Table 11.

Step 10: Analyze the benefit type and cost type criteria's normalized weighted values, as shown in Table 12.

Step 11: Determine the final rankings, as shown in Table 13.

Table 12 Total of all the values for the cost and benefit criterion

Cost type criteria	Benefit type criteria
0.1061	0.1390
0.0285	0.1157
0.0516	0.1289
0.0984	0.2442
0.0976	0.1793
0.0787	0.1560

Table 13 Final ranking values

Al_1	0.4922
Al_2	0.5020
Al_3	0.6522
Al_4	0.4541
Al_5	0.5053
Al_6	0.4676

Table 14 Comparison results

Authors	Methodologies	Rankings	Best alternative
Chen [58]	VIKOR	$Al_3 > Al_5 > Al_2 > Al_6 > Al_1 > Al_4$	Al_3
Ju et al. [59]	TODIM	$Al_3 > Al_2 > Al_5 > Al_1 > Al_6 > Al_4$	Al_3
Fan et al. [60]	COPRAS	$Al_3 > Al_1 > Al_2 > Al_5 > Al_6 > Al_4$	Al_3
Zhang and Wei [61]	D-CRITIC and CPT–CoCoSo	$Al_3 > Al_5 > Al_2 > Al_6 > Al_1 > Al_4$	Al_3
Ali [62]	CRITIC-MARCOS	$Al_3 > Al_4 > Al_2 > Al_6 > Al_5 > Al_1$	Al_3
Anafi et al. [63]	CRITIC-MAUT	$Al_3 > Al_5 > Al_2 > Al_6 > Al_1 > Al_4$	Al_3
Proposed		$Al_3 > Al_5 > Al_2 > Al_1 > Al_6 > Al_4$	Al_3

So the final ranking of alternatives is $Al_3 > Al_5 > Al_2 > Al_1 > Al_6 > Al_4$.

While the criteria and case study presented herein are specifically tailored to the strategic selection of VRS within the last-mile delivery sector in England, it is important to note that the mathematical framework of the proposed linear programming-based T-spherical fuzzy AROMAN model is inherently flexible. Its robust handling of uncertainty and multi-criteria evaluation makes it applicable to a wide range of complex decision-making scenarios across different industries, such as supply chain network design, renewable energy site selection, or healthcare resource management.

5.1 Comparative Analysis

As illustrated in Table 14, while several established methodologies such as VIKOR, TODIM, and CRITIC-MAUT converge on Al_3 as the optimal alternative, the proposed linear programming-based T-spherical fuzzy AROMAN method offers distinct technical advantages in its mathematical framework.

The superiority of the proposed approach is characterized by three primary factors:

1. **Advanced Uncertainty Modeling:** Unlike traditional fuzzy or intuitionistic sets, the T-spherical fuzzy logic utilized here employs a three-dimensional membership function-incorporating membership (M), abstention (N), and non-membership (L) degrees. This allows for a more granular representation of expert hesitation and neutral thoughts, which is critical in the high-stakes environment of LMD logistics.
2. **Optimization of Weight Information:** Many existing models (e.g., TODIM) struggle with "incomplete weight information," often relying on subjective estimations. Our methodology integrates a linear programming-based weight determination step (Step 6), which mathematically optimizes the weight vector ($\mathfrak{W}_i^{\gamma}$)

within defined constraints. This removes human bias and ensures the ranking is grounded in an optimized objective framework.

3. **Structural Robustness via Two-Step Normalization:** The AROMAN component utilizes a dual normalization process (linear and vector), which stabilizes the data against outliers and ensures that the final ranking (R_i) remains consistent even when criteria scales vary significantly.

The "superiority" of our model is not merely defined by the final selection of Al_3, but by the mathematical reliability and reduced subjectivity of the decision-making process itself. This makes the proposed method more resilient to the volatile and uncertain data typical of modern supply chain operations.

5.2 Managerial Implications

These empirical results have far-reaching managerial implications for LMD organizations aiming at maximizing their operational performance. The expression of an outlined procedure of utilizing the T-spherical fuzzy AROMAN approach to the identification of VRSs is beneficial in this respect. The framework realizes a step-by-step process of selection in line with organizational requirements. With the aid of such sophisticated mathematical modeling tools and fuzzy logic, the methodology provides managers with a logical framework to process the choice of VRS alternatives based on a plethora of criteria. The ability will enable LMD firms to overcome the complexity and uncertainty in the environment they operate in, which eventually aids in arriving at the best solution to VRS. The effectiveness and feasibility of the proposed methodology was proved on the basis of a practical case study performed in an LMD company located in England. In general, the study can provide significant information and tools that will allow LMD managers to remain efficient and competitive in terms of their operations within an ever-changing market.

5.3 Theoretical Limitations

The possible weakness of the proposed methodology is that the effective implementation of both the T-spherical fuzzy logic and the linear programming principles is a complicated process. This complexity can also cause long computation times and create serious computation difficulties. Additionally, the applicability of the methodology is limited because the case study has been limited to an LMD business in England, and thus, one can question whether such a methodology is applicable in different geographical and operational settings. The definition of specific standards and the choice of specialists can subject the process to subjectivity thus creating bias and inconsistency throughout the evaluation process. Practically, the implementation of such a method in the highly complicated situation of making decisions and extensive process of selecting VRS is laborious, especially in relation to data management and resources. Assumptions and uncertainties are inherent in the utilization of fuzzy logic technology and mathematical models and therefore the resultant decisions may be undermined in regard to their robustness and reliability. This means that operational environments and technological changes can impact the performance of the VRS and require regular changes and modifications, which would allow its practical applicability to continue. Real obstacles that could impede the actualization of the suggested algorithm include support of the stakeholders, organizational opposition, and lack of resources, which will destabilize the feasibility and sustainability of the suggested algorithm.

6 Conclusions

In conclusion, this paper introduces a pioneering method, the linear programming-based T-spherical fuzzy alternative ranking order method, aimed at aiding LMD companies in selecting advanced VRS. Given the established significance of VRS selection in enhancing operational efficacy and customer satisfaction, our study's principal aim is to empower LMD organizations to make informed decisions. Through a systematic approach that incorporates sophisticated mathematical modeling and fuzzy logic technology, this article presents a method for evaluating and selecting VRS alternatives based on diverse criteria. By integrating T-spherical fuzzy logic and linear programming principles, our method addresses the complexities and uncertainties inherent in LMD operations, facilitating businesses in identifying the most suitable VRS solution for their specific needs. Illustrated through a practical case study in an English LMD company, wherein a panel of experts establishes criteria and evaluates numerous VRS options, our proposed algorithm proves effective in determining the optimal VRS choice. This research contributes to the field of logistics and transportation management by offering robust methodologies for VRS selection, thereby enabling LMD companies to enhance their competitiveness and operational efficiency in dynamic market environments. Additional research should explore the implementation of the suggested technique in a broader range of geographical areas and various LMD scenarios to assess its effectiveness and applicability. Furthermore, an analysis of the possible incorporation of emerging technologies such as blockchain and artificial intelligence into VRS selection processes could provide LMD enterprises further operational efficiencies and enhanced decision-making capacities.

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Data Availability The data that support the findings of this study are included within the study.

Declarations

Competing Interests The authors declare no competing interests.

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