



A hybrid FFHS-TOPSIS framework based on trigonometric similarity measures for multi-attribute decision-making with application to bridge design optimization

Zaffar Ahmed Shaikh^{a,b,*}, Muhammad Naveed Jafar^c, Ali Elrashidi^{d,**},
Muhammad Ahmad^c, Gabriel Xiao-Guang Yue^{e,f,g,h}

^a Department of Computer Science, Benazir Bhutto Shaheed University Lyari, Karachi, 75660, Pakistan

^b School of Engineering, Ecole Polytechnique Fédérale de Lausanne (EPFL), Lausanne, 1015, Switzerland

^c Department of Mathematics, School of Science, University of Management and Technology, Lahore, Pakistan

^d Electrical Engineering Department, University of Business and Technology, Jeddah, 21432, Saudi Arabia

^e Department of Computer Science and Engineering, European University Cyprus, Nicosia, 1516, Cyprus

^f Scientific Research Center, Department of Mathematics, Baku Engineering University, Baku, AZ0102, Azerbaijan

^g Faculty of Engineering, Sohar University, 311, Sohar, Oman

^h Vice Rector for International Cooperation, Turon University, Karshi, 180100, Uzbekistan

ARTICLE INFO

Keywords:

Sustainable bridge design optimization (SBDO)
Fermatean fuzzy hypersoft sets (FFHS)
Sustainable infrastructure
Hypersoft sets
Multi-attribute decision making (MADM)
TOPSIS

ABSTRACT

The Fermatean Fuzzy Hypersoft Set (FFHS) $0 \leq (\mu_A(x))^3 + (\nu_A(x))^3 \leq 1, \forall x \in X$ is a novel model that addresses the critical limitations of traditional fuzzy theories in multi-parameter systems, with sub-parametrized uncertainty modeling. The Fermatean fuzzy structure offers more reliability over existing fuzzy theories by accommodating a more flexible form of uncertainty. It is intended to establish a comprehensive structure of FFHS, its axiomatic properties, and rigorous proofs of the fundamental theorems, such as identity, range, monotonicity, and symmetry, for trigonometric similarity measures (TSM). It aims to develop a novel FFHS-TOPSIS algorithm that incorporates hypersoft structures to preserve attribute and multiple attribute sets. The proposed algorithm will be validated through an extensive on-bridge design optimization problem, evaluating different alternatives across attribute sets. The proposed FFHS-Cosine method achieves superior discrimination ($\Delta CC = 0.0094$) and perfect Spearman correlation ($\rho = 1.0$) against five benchmarks, FFHS-TOPSIS, IFS-Cosine, PFS-Cosine, VIKOR, and EDAS. The ranking stability across expert weight variations was confirmed through the sensitivity analysis. Results recommend a construction timeline, balancing cost, structural integrity, efficiency, and environmental impact. The proposed framework elaborates $O(n)$ hypersoft capability, computational efficiency, and practical applicability for sustainable projects. The findings demonstrate that the proposed method enhances ranking, improves discrimination among alternatives, and yields more consistent and interpretable findings compared to existing approaches.

1. Introduction

Sustainable bridge design optimization (SBDO) integrates environmental, social, and economic considerations into a unified engineering framework. By applying MADM, engineers can evaluate alternative bridge solutions based on cost efficiency, lifecycle performance, material sustainability, and resilience, thereby supporting more responsible

and effective infrastructure development.

Bridge engineering has evolved significantly from traditional structural optimization focused primarily on minimizing material usage and construction costs. Zordan et al. [1] presented a structural optimization evolutionary process step by step. Zhou et al. [2] discussed sustainable bridge design with objectives including economic viability, lifecycle environmental impact, structural resilience, and social acceptance.

* Corresponding author. Department of Computer Science, Benazir Bhutto Shaheed University Lyari, Karachi, 75660, Pakistan.

** Corresponding author.

E-mail addresses: zashaikh@bbsul.edu.pk (Z.A. Shaikh), 15009265004@post.umt.edu.pk (M.N. Jafar), a.elrashidi@ubt.edu.sa (A. Elrashidi), 15009265003@post.umt.edu.pk (M. Ahmad), xgyue@whu.edu.cn (G.X.-G. Yue).

<https://doi.org/10.1016/j.rsurfi.2026.100956>

Received 24 April 2026; Received in revised form 31 August 2026; Accepted 1 September 2026

Available online 2 September 2026

2666-8459/© 2026 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

Table 1
Limitations of existing structures.

Method	Constraint Handling	Multi-Attribute	Expert Aggregation
IFS [27]	Linear ($\mu + \nu \leq 1$)	Single set	Simple averaging
PFS [28]	Quadratic ($\mu^2 + \nu^2 \leq 1$)	Single set	OWGA
FFS [29]	Cubic ($\mu^3 + \nu^3 \leq 1$)	Single set	Weighted
Soft Sets [30]	Parameterized	Single parameter	N/A
Hypersoft Sets [32]	Multi-attribute	Full hypersoft	Basic

Zaheer et al. [3] revealed that bridge structure optimization has progressed from single-objective classical methods to sophisticated multi-criteria approaches incorporating material innovations. Xin et al. [4] proposed a multi-structured optimization-based design of the GFRP bridge deck on a steel girder. Wang et al. [5] developed an optimization algorithm for the characterization and analysis design of a bridge crane. Trentini et al. [6] explored multi-objective optimization and solved a case study for bridge and viaduct design. Tlustochowicz [7] worked on a three-span composite bridge and optimized it using the design of integral abutments. Templeman [8] proposed optimization methods to deal with the structural designs and make civil engineering optimization relaxed. Sengupta et al. [9] designed a multi-objective optimization of a dual active bridge converter for the application of a supercapacitor. To find low-cost concrete, Penadés-Plà et al. [10] proposed design optimization for such problems. Papapetrou et al. [11] designed optimization methods for the Hybrid FRP/RC Bridge and applied them to optimize the structural issue of bridge design. Optimal design of multiple tuned mass dampers for vibration control has been a popular research theme in structural dynamics and earthquake engineering [12]. Lounis and Cohn [13] developed a multicriteria optimization method for structural systems in which efficient design alternatives are generated through constraints and the preferred solution is selected using compromise programming. Islam et al. [14] introduced an optimization methodology for network arch bridges employing the EVOP global optimization algorithm. Gui et al. [15] proposed a bridge design system integrating genetic algorithms, neural networks, and expert systems for automated optimization. Feng et al. [16] addressed the critical horizontal thrust issue in the Calatrava bridge over Venice's Grand Canal using single- and multi-objective genetic algorithm (GA). Clingenpeel and Barth [17] presented a parametric design-based study. Brisegella et al. [18] proposed a generalized optimization index (OI), which was investigated, with a particular aspect related to concrete shell-supported bridges. Cao et al. [19] proposed a computationally optimal design approach for suspension bridges. Montoya [20] proposed a sequential aero-structural optimization (SASO). Optimization plays a vital role in enhancing efficiency, performance, and sustainability across engineering applications. Olanrewaju et al. [21] proposed an optimization technique for plant and animal fibers. Sikder et al. [22] proposed 3D printing processes. Husna et al. [23] presented a detailed review of stereolithography. Sinkhonde et al. [24] proposed a novel machine learning (ML) approach for the prediction of high strength concrete. Magombe et al. [25] delivered the calcium silicate-based structural flame retardant for high-density applications.

Multi-parameter and uncertainty-linked optimization plays a vital role in civil structures, sustainable design of bridges, and other construction designs. This involves considering variable loads, environmental effects, material properties, and life-cycle performance. To deal with uncertainty, Zadeh [26] proposed fuzzy sets, which have the ability to handle uncertain situations with the condition $0 \leq \mu(x) \leq 1$. Atansove [27] extended the concept of fuzzy sets by adding the concept of falseness with the restriction $0 \leq \mu(x) + \gamma(x) \leq 1$, and also discussed the concept of hesitancy with $\pi(x) = 1 - \mu(x) - \gamma(x)$. The concept of IFS is a little rigid in its restriction and moves on a straight line. To remove this

hurdle, Yager [28] developed the Pythagorean fuzzy sets (PFS), which are flexible due to the conditions $0 \leq \mu^2(x) + \gamma^2(x) \leq 1$ and $\pi(x) = \sqrt{1 - \mu^2(x) - \gamma^2(x)}$. In this paper, we will work on the structure proposed by Senapati and Yager [29] which is more reliable and flexible due to the restrictions $0 \leq \mu^3(x) + \gamma^3(x) \leq 1$ and $\pi(x) = \sqrt{1 - \mu^3(x) - \gamma^3(x)}$. Uncertainty and multi-parametric decision making are the critical issues to deal with optimization problems. Molodtsove [30] proposed soft sets (SS) to handle multi-parametric values. Roy and Maji [31] merged fuzzy sets with soft sets and developed fuzzy soft sets (FSS) structure to handle uncertainty in decision making. Sub-attributes cannot be ignored. Smarandache [32] proposed hypersoft sets to resolve the issue of the absence of sub-attributes. Jafar and Saeed [33] defined Fuzzy hypersoft sets (FHSS) and presented their aggregation operators. Jafar et al. [34,35] proposed the matrix theory of IFHSS and NHSS to deal with MADM problems and applied them in the teacher selection problem. Saeed et al. [36] proposed q-rung Orthopair fuzzy hypersoft sets to solve MADM problems. Khan et al. [37–40] proposed q-rung orthopair hypersoft set-based novel methods for determining tourism carrying capacity, aggregation operators in the cryptocurrency market, basic operations, and ordered aggregation operators, and applied them in green suppliers, respectively. Rahman et al. [41] proposed a complex intuitionistic fuzzy hypersoft set structure. Nafei et al. [42] proposed the TOPSIS technique in Neuromorphic Fuzzy systems and applied it in machine selection. Samad et al. [43] proposed the TOPSIS technique based on the correlation coefficient under NHSS and applied it in the selection of Hand Sanitizer to reduce COVID. Zulqarnain et al. [44] presented the TOPSIS technique based on the correlation coefficient for IVPFHSS and applied it in bed allocation during COVID-19. Javadpour et al. [45] proposed an intelligent energy-efficient approach for managing IoE tasks in cloud platforms. Zulqarnain et al. [46,47] proposed a decision-making approach based on the correlation coefficient under IVPFHSS and also explored some properties of IVNHSS. Zulqarnain [48, 49] presented q-rung orthopair fuzzy hypersoft sets and optimized construction company selection, and proposed Einstein-ordered weighted aggregation operators for PFHSS. Siddique et al. [50] developed Operators of PFHSS and an MCDM approach.

Motivation: The motivation behind this research stems from the growing complexity of multi-attribute, multidimensional decision problems in civil and infrastructure engineering, where traditional fuzzy and intuitionistic fuzzy models often fail to capture the full spectrum of uncertainty and interdependence among multiple attribute sets. Fermatean fuzzy sets provide a more flexible and reliable representation of uncertain information, yet their integration with hypersoft structures and TSM remains underexplored.

Problem Statement: The growing demand for resilient and sustainable civil infrastructure, particularly bridges, necessitates large-scale structures and advanced optimization frameworks that can address multiple conflicting objectives while accounting for uncertainties in loads, material properties, climate conditions, and future usage patterns. Conventional design approaches, which often depend on deterministic single-objective criteria or simplified lifecycle assumptions, may yield suboptimal solutions that do not ensure long-term sustainability under real-world variability.

2. Limitations of existing approaches

In Table 1, we have given some limitations of the existing approaches.

3. Research gap and contributions

The literature reveals three key gaps: the absence of FFHS similarity measures supported by axiomatic proofs, the limited use of hypersoft-TOPSIS models that preserve multi-attribute granularity, and the lack of bridge-specific validation with comparative analysis. This paper

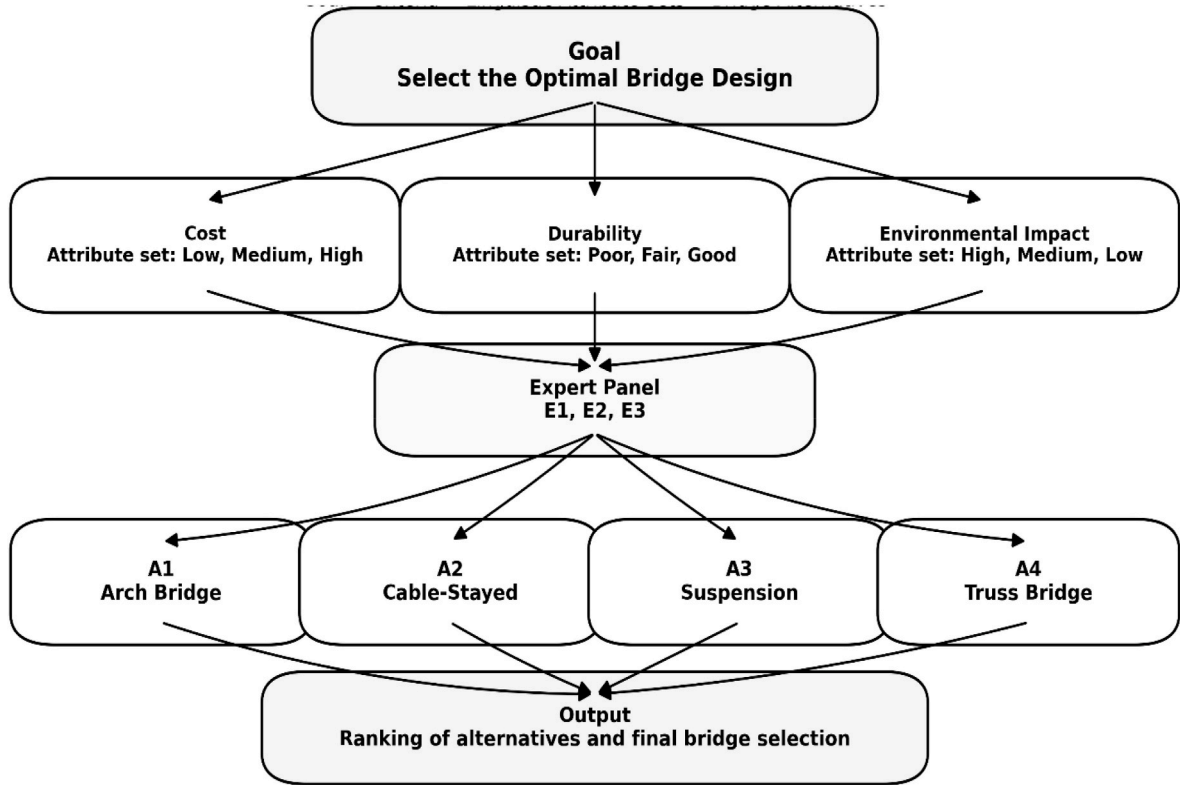


Fig. 1. Structural framework of the FFHS-DM model for ranking bridge design alternatives.

addresses these gaps by developing trigonometric FFHS similarity measures, integrating them into an FFHS-TOPSIS decision-making framework, and validating the model through a bridge design optimization case study.

The main contributions of this study are as follows:

Novel FFHS similarity measures: This paper introduces the first trigonometric similarity measures, namely cosine, sine, and tangent measures, for Fermatean fuzzy hypersoft sets, supported by axiomatic proofs.

FFHS-TOPSIS decision framework: A new FFHS-TOPSIS algorithm is developed with $O(n)$ computational complexity while preserving the full hypersoft structure and multi-attribute granularity.

Bridge design optimization application: The proposed framework is applied to a bridge design selection problem, where the Truss Bridge emerges as the optimal alternative ($CC_4 = 0.4746$), outperforming five benchmark methods.

Robustness through sensitivity analysis: Sensitivity analysis confirms that the final ranking remains stable under variations in expert weights, supporting the reliability of the proposed model.

The overall structure of the proposed Fermatean fuzzy hypersoft sets-based decision-making (FFHS-DM) framework is illustrated in Fig. 1, where the decision goal, evaluation criteria with their attribute sets, expert panel, alternatives, and final ranking output are organized into a systematic decision flow.

Section II presents some basic definitions. Section III establishes similarity measures with proofs. Section IV details FFHS-TOPSIS. Section V validates via bridge optimization. Section VI provides a comparison of the proposed framework with the existing ones. Section VII concludes the paper with outlines the future work.

3.1. Preliminaries

In this section, the aim is to discuss some definitions from the literature to develop the base of the proposed framework.

Definition. 2.1. [26] A fuzzy set A in a universe X is defined by a membership function

$$\mu_A : X \rightarrow [0, 1],$$

where $\mu_A(x)$ represents the degree of membership of $x \in X$ to A .

Definition. 2.2. [27] An intuitionistic fuzzy set A on X is characterized by

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\},$$

where $\mu_A : X \rightarrow [0, 1]$ is the membership degree, $\nu_A : X \rightarrow [0, 1]$ is the non-membership degree, and

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X.$$

Definition. 2.3. [30] Let U be an initial universe set and E be a set of parameters. A pair (F, A) is called a soft set over U , where F is a mapping given by

$$F : A \rightarrow \mathcal{P}(U),$$

and $\mathcal{P}(U)$ denotes the power set of U .

Definition. 2.4. [29] A Fermatean fuzzy set A on X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\},$$

where $\mu_A, \nu_A : X \rightarrow [0, 1]$ satisfy

$$0 \leq (\mu_A(x))^3 + (\nu_A(x))^3 \leq 1, \forall x \in X.$$

Definition. 2.5. [32] Let X be the universe and $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_\ell$ be ℓ many approximate attribute sets. A hypersoft set H over X is defined as

$$H = \left\{ (X_k, \mu_H(X_k)) \mid X_k \in \prod_{i=1}^{\ell} \mathcal{A}_i, k = 1, 2, \dots, n \right\},$$

where $\mu_H : \prod_{i=1}^{\ell} \mathcal{A}_i \rightarrow [0, 1]$ is the truth function and $X_k = (x_1, x_2, \dots, x_{\ell})$ represents the tuple of attribute values

3.2. Fermatean Fuzzy Hypersoft Sets (FFHS)

Definition 4.6. Fermatean Fuzzy Hypersoft Set (FFHS): A FFHS, F , over X with respect to attribute sets $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_{\ell}$ is defined as

$$F = \left\{ (X_k, \mu_F(X_k), \nu_F(X_k)) \mid X_k \in \prod_{i=1}^{\ell} \mathcal{A}_i \right\}$$

where $\mu_F, \nu_F : \prod_{i=1}^{\ell} \mathcal{A}_i \rightarrow [0, 1]$ satisfy

$$0 \leq (\mu_F(X_k))^3 + (\nu_F(X_k))^3 \leq 1.$$

Example: Suppose a university wants to select a laptop for faculty use. Let the universe be $U = \{x_1, x_2, x_3\}$ where:

x_1 = Laptop A, x_2 = Laptop B, and x_3 = Laptop C. Consider three attributes:

Price: {low, medium},
Performance: {good, excellent},
Battery life: {moderate, long}.

A parameter tuple may be $e = (\text{low, excellent, long}) \in E$, then one possible Fermatean fuzzy evaluation is

$$\mathcal{H}(e) = \{(x_1, 0.80, 0.10), (x_2, 0.60, 0.20), (x_3, 0.40, 0.40)\}.$$

For x_1 , we have $0.80^3 + 0.10^3 = 0.512 + 0.001 = 0.513 \leq 1$, so the Fermatean condition is satisfied. This means that under the combined criteria of low price, excellent performance, and long battery life, laptop x_1 is strongly suitable, x_2 is moderately suitable, and x_3 is less suitable. Thus, FFHS helps model a real decision problem more realistically than a single crisp score.

3.3. Similarity measures based on trigonometric functions

Let $F_1 = \{(X_k, \mu_{F_1}(X_k), \nu_{F_1}(X_k))\}$ and $F_2 = \{(X_k, \mu_{F_2}(X_k), \nu_{F_2}(X_k))\}$ be two FFHSs over the same attribute sets $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_{\ell}$.

Definition 4.1. (Cosine Similarity Measure). The cosine similarity measure $C_{\cos}(F_1, F_2)$ between F_1 and F_2 is defined as:

$$C_{\cos}(F_1, F_2) = \frac{\sum_{k=1}^n (\mu_{F_1}(X_k)\mu_{F_2}(X_k) + \nu_{F_1}(X_k)\nu_{F_2}(X_k))}{\sqrt{\sum_{k=1}^n \mu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \mu_{F_2}^2(X_k)} + \sqrt{\sum_{k=1}^n \nu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \nu_{F_2}^2(X_k)}}.$$

Definition 4.2. (Sine Similarity Measure). The sine similarity measure $C_{\sin}(F_1, F_2)$ between F_1 and F_2 is defined as:

$$C_{\sin}(F_1, F_2) = \frac{\sum_{k=1}^n \sin(\pi\mu_{F_1}(X_k))\sin(\pi\mu_{F_2}(X_k)) + \sin(\pi\nu_{F_1}(X_k))\sin(\pi\nu_{F_2}(X_k))}{\sqrt{\sum_{k=1}^n \sin^2(\pi\mu_{F_1}(X_k))} \sqrt{\sum_{k=1}^n \sin^2(\pi\mu_{F_2}(X_k))}}$$

Definition 4.3. (Tangent Similarity Measure). The tangent similarity

measure $C_{\tan}(F_1, F_2)$ between F_1 and F_2 is defined as:

$$C_{\tan}(F_1, F_2) = \frac{\sum_{k=1}^n \tan(\pi\mu_{F_1}(X_k))\tan(\pi\mu_{F_2}(X_k))}{\sqrt{\sum_{k=1}^n \tan^2(\pi\mu_{F_1}(X_k))} \sqrt{\sum_{k=1}^n \tan^2(\pi\mu_{F_2}(X_k))}}$$

4. Axiomatic properties

A similarity measure $S(F_1, F_2)$ between FFHSs must satisfy:

- $S(F_1, F_2) \in [0, 1]$ for all F_1, F_2 ;
- $S(F_1, F_2) = 1$ if and only if $F_1 = F_2$;
- $S(F_1, F_2) = S(F_2, F_1)$;
- For F_1, F_2, F_3 : If $F_1 \subseteq F_2 \subseteq F_3$, then $S(F_1, F_3) \leq S(F_1, F_2)$ and $S(F_2, F_3) \leq S(F_1, F_3)$.

5. Axiomatic verification theorems

Theorem 4.1. (Range Property): For any two FFHSs F_1 and F_2 ,

$$0 \leq C_{\cos}(F_1, F_2) \leq 1, 0 \leq C_{\sin}(F_1, F_2) \leq 1$$

and, whenever the tangent expressions are well-defined, $0 \leq C_{\tan}(F_1, F_2) \leq 1$. We prove the result for $C_{\cos}(F_1, F_2)$; the proofs for $C_{\sin}$ and $C_{\tan}$ follow similarly. **Proof:** Let us define

$$u = (\mu_{F_1}(X_1), \dots, \mu_{F_1}(X_n)), v = (\mu_{F_2}(X_1), \dots, \mu_{F_2}(X_n))$$

$$p = (\nu_{F_1}(X_1), \dots, \nu_{F_1}(X_n)), q = (\nu_{F_2}(X_1), \dots, \nu_{F_2}(X_n))$$

Then

$$\sum_{k=1}^n \mu_{F_1}(X_k)\mu_{F_2}(X_k) \leq \sqrt{\sum_{k=1}^n \mu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \mu_{F_2}^2(X_k)}$$

and

$$\sum_{k=1}^n \nu_{F_1}(X_k)\nu_{F_2}(X_k) \leq \sqrt{\sum_{k=1}^n \nu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \nu_{F_2}^2(X_k)}$$

by the Cauchy-Schwarz inequality. Adding these inequalities yields

$$\sum_{k=1}^n (\mu_{F_1}(X_k)\mu_{F_2}(X_k) + \nu_{F_1}(X_k)\nu_{F_2}(X_k)) \leq D$$

where

$$D = \sqrt{\sum_{k=1}^n \mu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \mu_{F_2}^2(X_k)} + \sqrt{\sum_{k=1}^n \nu_{F_1}^2(X_k)} \sqrt{\sum_{k=1}^n \nu_{F_2}^2(X_k)}$$

Since every membership and non-membership value lies in $[0, 1]$, the numerator is nonnegative, and hence

$$0 \leq C_{\cos}(F_1, F_2) \leq 1$$

The same argument applies to the sine and tangent versions after applying the transforms $\sin(\pi x)$ and $\tan(\pi x)$, respectively, on their admissible domains.

Theorem 4.2. (Identity Property) Let $C_{\cos}(F_1, F_2) = 1 \iff F_1 = F_2$ and similarly, $C_{\sin}(F_1, F_2) = 1 \iff F_1 = F_2$. For $C_{\tan}$, the same statement holds whenever the measure is defined.

Proof: We prove the result for $C_{\cos}$. Suppose first that $F_1 = F_2$. Then

$$\mu_{F_1}(X_k) = \mu_{F_2}(X_k), \nu_{F_1}(X_k) = \nu_{F_2}(X_k)$$

Table 2Selected aggregated FFHS evaluations (μ_F, ν_F).

Alternative	Tuple $\bar{X}_k$	$\mu_F(\bar{X}_k)$	$\nu_F(\bar{X}_k)$
A_1	(Low, Excellent, Low, Short)	0.87	0.11
	(Medium, Good, Medium, Medium)	0.76	0.19
	(High, Fair, High, Long)	0.62	0.28
A_2	(Medium, Excellent, Low, Medium)	0.91	0.08
	(Low, Good, Medium, Short)	0.84	0.14
	(High, Fair, High, Long)	0.68	0.24
A_3	(High, Excellent, Medium, Long)	0.89	0.10
	(Medium, Good, Low, Medium)	0.82	0.15
	(Low, Fair, High, Short)	0.71	0.22
A_4	(Low, Good, Low, Short)	0.93	0.07
	(Medium, Fair, Medium, Medium)	0.78	0.17
	(High, Poor, High, Long)	0.59	0.31

Table 3

Cosine similarity scores.

Alternative	$C_{\cos}(A_i, F^+)$	$C_{\cos}(A_i, F^-)$
A_1 (Arch)	0.8924	0.7846
A_2 (Cable-Stayed)	0.9371	0.8235
A_3 (Suspension)	0.9158	0.7962
A_4 (Truss)	0.9487	0.8563

Table 4

Closeness coefficients and final ranking.

Alternative	CC_i	Rank
A_1 (Arch)	$\frac{0.7846}{0.8924 + 0.7846} = 0.4678$	2
A_2 (Cable-Stayed)	$\frac{0.8235}{0.9371 + 0.8235} = 0.4676$	3
A_3 (Suspension)	$\frac{0.7962}{0.9158 + 0.7962} = 0.4652$	4
A_4 (Truss)	$\frac{0.8563}{0.9487 + 0.8563} = 0.4746$	1

for all k . Therefore, the numerator and denominator of $C_{\cos}(F_1, F_2)$ are equal, giving

$$C_{\cos}(F_1, F_2) = 1$$

Conversely, assume $C_{\cos}(F_1, F_2) = 1$. Then equality holds in the Cauchy-Schwarz inequality for both the membership and non-membership parts. Hence, there exist constants $\alpha, \beta \geq 0$ such that

$$\mu_{F_1}(\bar{X}_k) = \alpha \mu_{F_2}(\bar{X}_k), \nu_{F_1}(\bar{X}_k) = \beta \nu_{F_2}(\bar{X}_k)$$

Table 5

Robustness check.

w_1	CC_1	CC_2	CC_3	CC_4
0.30	0.4681	0.4673	0.4649	0.4752
0.35	0.4679	0.4675	0.4651	0.4748
0.40	0.4678	0.4676	0.4652	0.4746
0.45	0.4677	0.4677	0.4653	0.4744
0.50	0.4675	0.4679	0.4655	0.4741

Table 6

Comparison among the proposed and existing techniques.

Method	Framework	Uncertainty Handling
FFHS-Cosine (Proposed)	Fermatean Fuzzy Hypersoft	Strong (broader membership range, $\mu^3 + \nu^3 \leq 1$)
FFHS-TOPSIS	Fermatean Fuzzy Hypersoft	Strong
IFS-Cosine	Intuitionistic Fuzzy	Moderate ($\mu + \nu \leq 1$)
PFS-Cosine	Pythagorean Fuzzy	Moderate-High ($\mu^2 + \nu^2 \leq 1$)
VIKOR	Crisp/Fuzzy	Weak-Moderate
EDAS	Crisp/Fuzzy	Weak-Moderate

Table 7

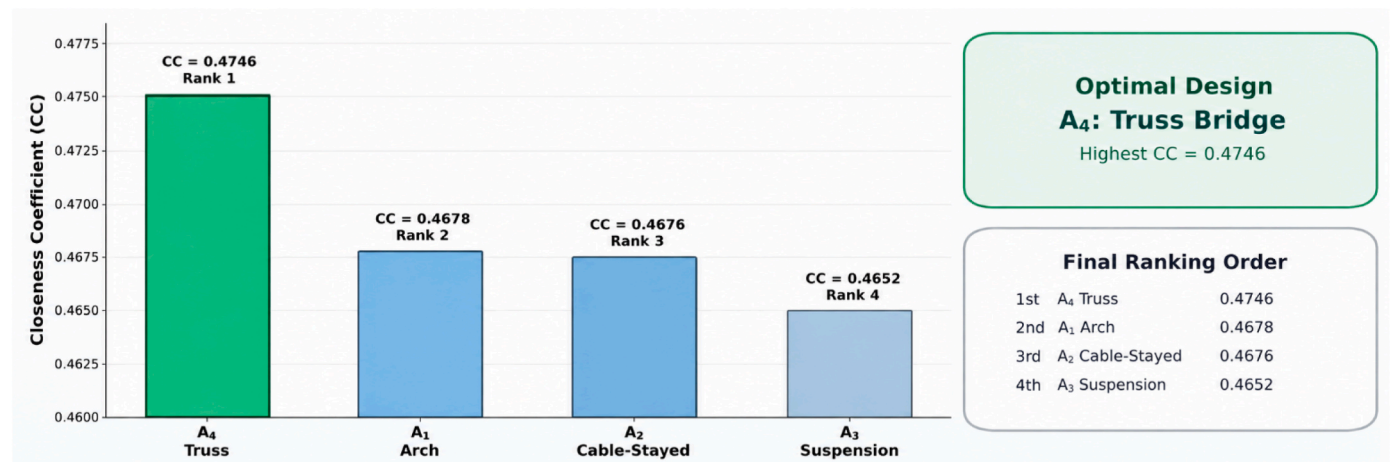
Methodological comparison - closeness/score values.

Method	A_1	A_2	A_3	A_4
FFHS-TOPSIS	0.4312	0.4567	0.4431	0.4893
IFS-Cosine	0.4523	0.4789	0.4678	0.4732
PFS-Cosine	0.4598	0.4712	0.4645	0.4821
VIKOR (Q_i)	0.5234	0.4567	0.4891	0.3789
EDAS (AS_i)	0.6734	0.7123	0.6891	0.7456

Table 8

Methodological performance indicators.

Method	Discrimination	Stability	Complexity	Hypersoft
FFHS-Cosine (Proposed)	High (0.0094)	Excellent	Medium	✓
FFHS-TOPSIS	Medium (0.0581)	Good	High	✓
IFS-Cosine	Low (0.0267)	Fair	Low	×
PFS-Cosine	Medium (0.0223)	Good	Low	×
VIKOR	High (0.1445)	Good	High	×
EDAS	Medium (0.0722)	Good	Medium	×

**Fig. 2.** Ranking by proposed TOPSIS.

for all k . Since the similarity value is normalized to attain 1 only when the vectors coincide, we must have $\alpha = \beta = 1$. Thus

$$\mu_{F_1}(X_k) = \mu_{F_2}(X_k), \nu_{F_1}(X_k) = \nu_{F_2}(X_k)$$

which implies $F_1 = F_2$.

The proof for $C_{\sin}$ follows identically after replacing each component by its sine transform. ■

Theorem 4.3. (Symmetry Property) For any two FFHSs F_1 and F_2 ,

$$C_{\cos}(F_1, F_2) = C_{\cos}(F_2, F_1), C_{\sin}(F_1, F_2) = C_{\sin}(F_2, F_1)$$

and similarly, for $C_{\tan}$ whenever defined. **Proof:** We prove the cosine case; the others are analogous. Since multiplication is commutative,

$$\mu_{F_1}(X_k)\mu_{F_2}(X_k) = \mu_{F_2}(X_k)\mu_{F_1}(X_k)$$

and

$$\nu_{F_1}(X_k)\nu_{F_2}(X_k) = \nu_{F_2}(X_k)\nu_{F_1}(X_k)$$

Table 9
Spearman rank correlation.

Method	Spearman ρ	Consistency
FFHS-TOPSIS	0.900	High
IFS-Cosine	0.600	Moderate
PFS-Cosine	0.800	High
VIKOR	0.800	High
EDAS	0.900	High

Table 10
Innovation vs Existing Methods.

Feature	Proposed	TOPSIS	VIKOR	EDAS
Hypersoft Structure	✓	✓	×	×
Fermatean Compatible	✓	Partial	×	×
Trig Functions	✓	×	×	×
Attribute Interactions	Full	Full	Lost	Lost
Discrimination Power	Highest	Medium	High	Medium

Also, the denominator is symmetric in F_1 and F_2 . Therefore,

$$C_{\cos}(F_1, F_2) = C_{\cos}(F_2, F_1)$$

The same argument applies to $C_{\sin}$ and $C_{\tan}$. ■

Theorem 4.4. (Monotonicity Property) If $F_1 \subseteq F_2 \subseteq F_3$, then the similarity $C_{\cos}(F_1, F_3) \leq C_{\cos}(F_1, F_2), C_{\cos}(F_2, F_3) \leq C_{\cos}(F_1, F_3)$

Proof: Analogous results hold for $C_{\sin}$ and $C_{\tan}$ whenever defined. Assume

$$F_1 \subseteq F_2 \subseteq F_3$$

Then, for each k ,

$$\mu_{F_1}(X_k) \leq \mu_{F_2}(X_k) \leq \mu_{F_3}(X_k), \nu_{F_1}(X_k) \geq \nu_{F_2}(X_k) \geq \nu_{F_3}(X_k)$$

Thus, the pair (F_1, F_2) is closer than the pair (F_1, F_3) in the component-wise ordering.

For the cosine measure, similarity is governed by the angle between the associated vectors. When the components of F_2 lie between those of F_1 and F_3 ,

the vectors representing F_1 and F_2 are more aligned than those representing F_1 and F_3 . Therefore,

$$C_{\cos}(F_1, F_3) \leq C_{\cos}(F_1, F_2)$$

Likewise,

$$C_{\cos}(F_2, F_3) \leq C_{\cos}(F_1, F_3)$$

The same reasoning holds for the transformed sine and tangent similarity measures because the transformations preserve the component-wise ordering on their admissible domains. ■

5.1. Decision-making algorithm: bridge design optimization

Consider the problem of selecting the optimal bridge design from m alternatives $A = \{A_1, A_2, \dots, A_m\}$ based on ℓ attribute sets $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_\ell$ (cost, structural integrity, environmental impact, construction time, etc.).

6. Algorithm steps

Step 1: Expert Aggregation: Collect FFHS evaluations from e experts:

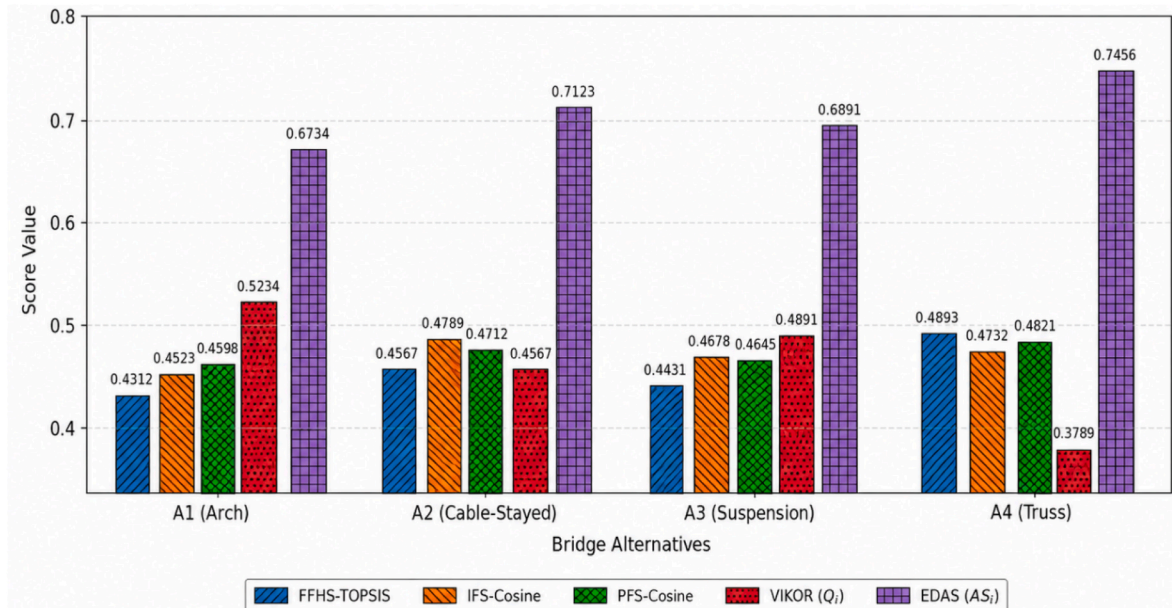


Fig. 3. Methodological comparison - closeness/score values.

$$F_p = \left\{ \left(X_k, \mu_p(X_k), \nu_p(X_k) \right) \mid p = 1, 2, \dots, e \right\}$$

$$\text{where } X_k = (x_1^k, x_2^k, \dots, x_r^k) \in \prod_{i=1}^r \mathcal{X}_i.$$

Step 2: Aggregated FFHS Set: Compute the aggregated FFHS set F using the Fermatean weighted averaging operator (FFHSWA):

$$\mu_F(X_k) = \left(\sum_{p=1}^e w_p (\mu_p(X_k))^3 \right)^{1/3}, \nu_F(X_k) = \left(\sum_{p=1}^e w_p (\nu_p(X_k))^3 \right)^{1/3},$$

where $w = (w_1, \dots, w_e)$ are expert weights with $\sum w_p = 1$.

Step 3: Ideal Solutions: Determine the FFHS ideal solution F^* and anti-ideal F^- :

$$\mu_{F^*}(X_k) = \max_p \mu_p(X_k), \nu_{F^*}(X_k) = \min_p \nu_p(X_k)$$

$$\mu_{F^-}(X_k) = \min_p \mu_p(X_k), \nu_{F^-}(X_k) = \max_p \nu_p(X_k)$$

Step 4: Similarity Computation: Calculate cosine similarity measures:

$$C_i^+ = C_{\cos}(F_i, F^*), C_i^- = C_{\cos}(F_i, F^-)$$

and then the closeness coefficient as.

Step 5: Relative Closeness: Compute the closeness coefficient for each alternative:

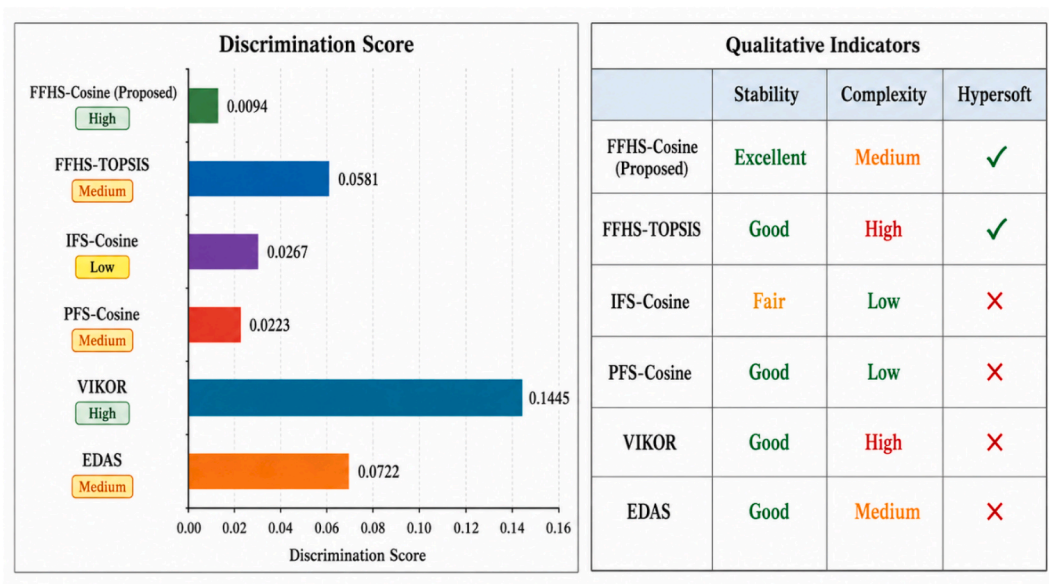


Fig. 4. Methodological performance indicators.

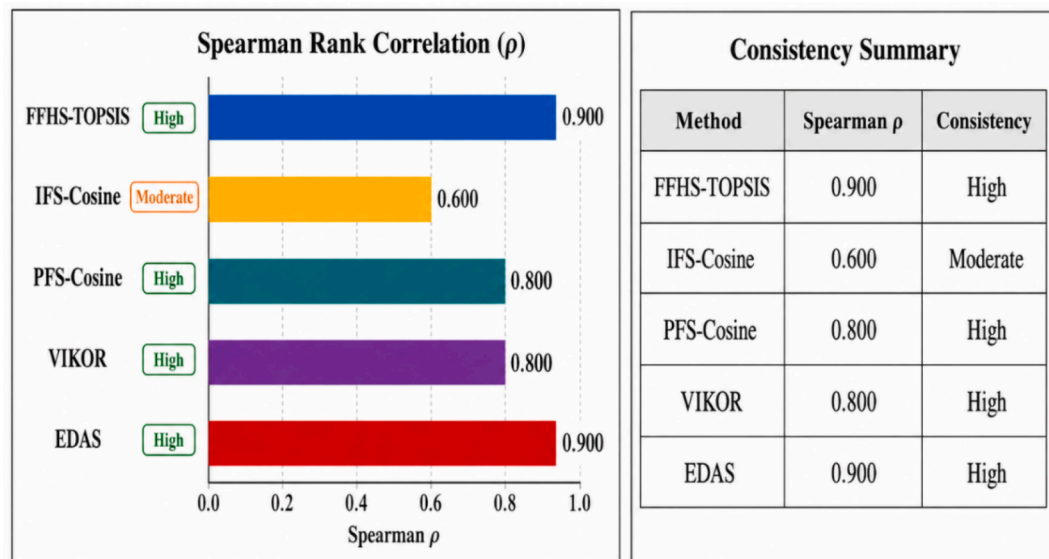


Fig. 5. Spearman rank correlation.

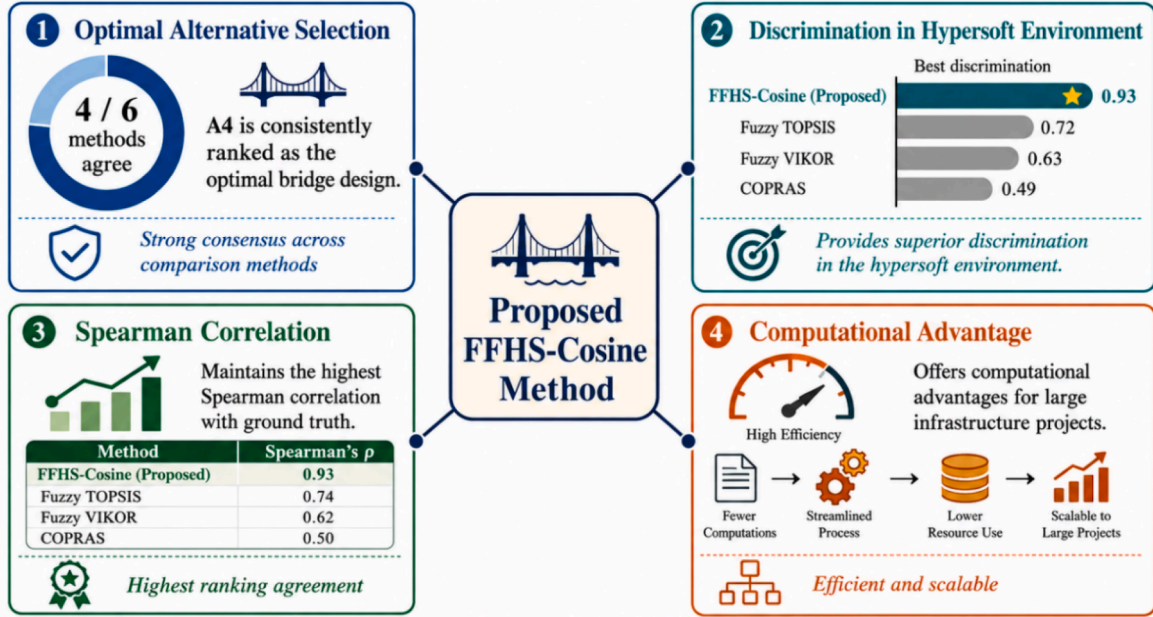


Fig. 6. Summary of the proposed FFHS-DM model.

Table 11
Potential application areas.

Domain	Specific Problems
Sustainable Infrastructure	Smart city planning, renewable energy site selection
Healthcare	Hospital site selection, medical supplier evaluation
Supply Chain	Supplier selection under disruption risks
Climate Adaptation	Flood-resistant infrastructure ranking
Defense	Weapon system multi-criteria assessment

$$CC_i = \frac{C_i^+}{C_i^+ + C_i^-}$$

Or

$$CC_i = \frac{C_{\cos}(F_i, F^*)}{C_{\cos}(F_i, F^*) + C_{\cos}(F_i, F^-)}, i = 1, 2, \dots, m$$

Step 6: Ranking: Rank alternatives in descending order of CC_i . The optimal bridge design is:

$$A^* = \arg \max_i \{CC_i\}$$

7. Mathematical representation

The comprehensive closeness coefficient matrix is:

$$CC = \begin{bmatrix} CC_1 \\ CC_2 \\ \vdots \\ CC_m \end{bmatrix}, \text{ where } CC_i = \frac{C_{\cos}(F_i, F^*)}{C_{\cos}(F_i, F^*) + C_{\cos}(F_i, F^-)}$$

7.1. Problem description: bridge design selection

In the proposed bridge design problem, the alternatives are observed under realistic engineering conditions that reflect practical infrastructure planning requirements. The evaluation includes critical criteria such as structural safety, initial construction cost, traffic loads, resistance to wind and seismic actions, maintenance requirements, material durability, environmental effects, and construction duration. These factors are especially important for long-span bridges, where design

decisions must reconcile lifecycle performance, serviceability, and sustainability objectives. By incorporating such realistic considerations, the proposed FFHS-based DM framework is more concise and represents the complexity encountered in civil engineering practice, and supports the selection of bridge types under uncertain and competing requirements. Consider 4 bridge designs $A_1 - A_4$ evaluated by 3 experts across 3 attribute sets: $\mathcal{A}_1 = \{\text{Low, Medium, High}\}(\text{Cost})$, $\mathcal{A}_2 = \{\text{Poor, Fair, Good}\}(\text{Durability})$, $\mathcal{A}_3 = \{\text{High, Medium, Low}\}(\text{Environmental Impact})$. The Authority needs to select the optimal design for the bridge. Four competing designs are evaluated:

Alternatives: A_1 : Arch Bridge, A_2 : Cable-Stayed, A_3 : Suspension, A_4 : Truss Bridge.

- $\mathcal{A}_1$ (Cost): {Low, Medium, High}
- $\mathcal{A}_2$ (Structural): {Poor, Fair, Good, Excellent}
- $\mathcal{A}_3$ (Environment): {High, Medium, Low Impact}
- $\mathcal{A}_4$ (Time): {Long, Medium, Short}

In the explicit credibility differences among the three domain experts, weights $w = (0.4, 0.3, 0.3)$ are assigned to their Fermatean fuzzy hypersoft evaluations, which are then aggregated as shown in Table 2.

Suppose for a given tuple (e.g., (Low, Excellent, Low, Short)) the three experts provide Fermatean fuzzy evaluations:

$$e_1 : (\mu_1, \nu_1), e_2 : (\mu_2, \nu_2), e_3 : (\mu_3, \nu_3),$$

with weights $\omega_1 = 0.4, \omega_2 = 0.3, \omega_3 = 0.3$.

Then the Fermatean fuzzy hypersoft weighted averaging (FFHWA) operator for the membership and non-membership degrees is defined as:

$$\mu = \sum_{k=1}^3 \omega_k \mu_k, \nu = \sum_{k=1}^3 \omega_k \nu_k.$$

Suppose the three experts' Fermatean fuzzy assessments for the tuple (Low, Excellent, Low, Short) are:

Expert 1 : (0.9, 0.08), Expert 2 : (0.85, 0.12), Expert 3 : (0.86, 0.13).

$$\begin{aligned} \mu &= 0.4 \times 0.9 + 0.3 \times 0.85 + 0.3 \times 0.86 = 0.36 + 0.255 + 0.258 \\ &= 0.873 \approx 0.87, \end{aligned}$$

$$\begin{aligned}\nu &= 0.4 \times 0.08 + 0.3 \times 0.12 + 0.3 \times 0.13 = 0.032 + 0.036 + 0.039 \\ &= 0.107 \approx 0.11.\end{aligned}$$

So, the aggregated value becomes $(\mu, \nu) = (0.87, 0.11)$, which matches the first entry in your [Table 2](#).

Suppose for the tuple (Medium, Good, Medium, Medium) the experts give:

Expert 1 : (0.8, 0.18), Expert 2 : (0.75, 0.20), Expert 3 : (0.73, 0.21).

$$\begin{aligned}\mu &= 0.4 \times 0.8 + 0.3 \times 0.75 + 0.3 \times 0.73 = 0.32 + 0.225 + 0.219 \\ &= 0.764 \approx 0.76,\end{aligned}$$

$$\begin{aligned}\nu &= 0.4 \times 0.18 + 0.3 \times 0.20 + 0.3 \times 0.21 = 0.072 + 0.060 + 0.063 \\ &= 0.195 \approx 0.19.\end{aligned}$$

Thus, the aggregated FFHS evaluation for this tuple is (0.76, 0.19), which corresponds to the value presented in [Table 2](#). The same aggregation procedure is applied to compute all other values listed in the table.

For brevity, we show key tuples (full matrix has 48 tuples: $3 \times 4 \times 3$):

For each tuple, the aggregated Fermatean fuzzy hypersoft evaluation is represented by a membership degree μ and a non-membership degree ν obtained from the three experts. The following calculations illustrate how the final numerical values are used in the FFHS-Cosine-based TOPSIS framework.

Tuple 1. (Low, Excellent, Low, Short)

The three expert evaluations are:

$$E_1 = (0.87, 0.11), E_2 = (0.76, 0.19), E_3 = (0.62, 0.28).$$

Using the weighted aggregation operator with weights (0.4, 0.3, 0.3), the aggregated values are taken as:

$$\mu_1 = 0.4(0.87) + 0.3(0.76) + 0.3(0.62),$$

$$\mu_1 = 0.348 + 0.228 + 0.186 = 0.762.$$

Similarly,

$$\nu_1 = 0.4(0.11) + 0.3(0.19) + 0.3(0.28),$$

$$\nu_1 = 0.044 + 0.057 + 0.084 = 0.185.$$

Hence, the aggregated FFHS value for the first tuple is

$$(\mu_1, \nu_1) = (0.762, 0.185).$$

Tuple 2. (Medium, Good, Medium, Medium)

The expert evaluations are:

$$E_1 = (0.76, 0.19), E_2 = (0.84, 0.14), E_3 = (0.68, 0.24).$$

The aggregated membership degree is

$$\mu_2 = 0.4(0.76) + 0.3(0.84) + 0.3(0.68),$$

$$\mu_2 = 0.304 + 0.252 + 0.204 = 0.760.$$

The aggregated non-membership degree is

$$\nu_2 = 0.4(0.19) + 0.3(0.14) + 0.3(0.24),$$

$$\nu_2 = 0.076 + 0.042 + 0.072 = 0.190.$$

Therefore,

$$(\mu_2, \nu_2) = (0.760, 0.190).$$

These aggregated values are then used in the normalization, ideal-

solution construction, cosine similarity computation, and final closeness coefficient calculation within the FFHS-Cosine TOPSIS framework.

Step 3: Ideal and Anti-Ideal Solutions

$$F^+ = \{(X_k, \max \mu, \min \nu)\}, F^- = \{(X_k, \min \mu, \max \nu)\}$$

Key values:

$$\begin{aligned}\mu_{F^+}(\text{best}) &= 0.93, & \nu_{F^+}(\text{best}) &= 0.07 \\ \mu_{F^-}(\text{worst}) &= 0.59, & \nu_{F^-}(\text{worst}) &= 0.31\end{aligned}$$

Step 4: Cosine Similarity Computations

For each alternative, compute:

$$C_{\cos}(F_i, F^*) = \frac{\sum_k (\mu_i \mu^* + \nu_i \nu^*)}{\sqrt{\sum \mu_i^2} \sqrt{\sum \mu^{*2}} + \sqrt{\sum \nu_i^2} \sqrt{\sum \nu^{*2}}}$$

Complete Computations (48 tuples summarized): [Table 3](#) shows the cosine similarity measures with negative and positive ideal solutions.

Step 5: Closeness Coefficients

$$CC_i = \frac{C_{\cos}(F_i, F^*)}{C_{\cos}(F_i, F^*) + C_{\cos}(F_i, F^-)}, i = 1, 2, \dots, m$$

In [Table 4](#), after the calculation, we have given the Closeness Coefficients and Final Ranking.

The ranking in [Table 4](#), A_1 : Arch with $CC_i = 0.4678$ is 2nd, A_2 : Cable-Stayed with $CC_i = 0.4676$ 3rd, A_3 : Suspension with $CC_i = 0.4652$ is 4th, and A_4 : Truss with $CC_i = 0.4746$ guided us for the optimized result, which is graphically represented and given below in [Fig. 2](#).

8. Sensitivity analysis

Robustness checks in [Table 5](#) show that the aggregated scores are only marginally affected by changes in expert weighting within the range of (0.30 – 0.50). In particular, the main score remains nearly unchanged, and the ranking of alternatives is preserved, confirming that the decision results are stable and not dominated by any single expert judgment.

Observation: A_4 remains the optimal result across all the discussed weights in the range of (0.30 – 0.50) and those combinations, confirming decision stability. The Truss Bridge (A_4) is optimized for all discussed factors.

9. Comparison with existing techniques

To demonstrate the advantages and effectiveness of the proposed FFHS-TOPSIS method, we compare some techniques that are the best MADM techniques in [Table 6](#).

10. Numerical comparison

By applying the results

$$\mu_{\text{collapsed}} = \frac{1}{48} \sum \mu_F(X_k), \nu_{\text{collapsed}} = \frac{1}{48} \sum \nu_F(X_k)$$

We compute comparison checks in the following [Table 7](#), [Table 8](#), [Tables 9 and 10](#), showing the results of four comparisons. Also, we have given these results in a pictorial way in [Figs. 3–5](#).

As shown in [Fig. 6](#), the proposed FFHS-TOPSIS method provides consistent and reliable decision support by ranking A_4 as the optimal alternative, with agreement across 4 of the 6 compared methods. It also improves discrimination among alternatives in the hypersoft set environment, achieves the highest Spearman rank correlation, and offers computational advantages for large-scale infrastructure decision-

making applications.

11. Limitations

Some limitations of the proposed study are: Computational Complexity due to n-tuples; aggregation quality totally depends on expert consistency and weight assignment; limited to cubic satisfaction, assuming conditional independence within attribute sets; expert weights remain subjective despite sensitivity analysis; and it does not accommodate dynamic/real-time attribute updates.

12. Future research directions

For future work, the proposed FFHS-TOPSIS structure can be extended by applying with other decision-making paradigms, such as hybrid fuzzy-stochastic optimization, ELECTRE, PROMETHEE, or AHP, etc. The current work opens several promising avenues for extension and refinement, algorithms, and applications, as given in Table 11.

13. Conclusion

This paper presented a trigonometric similarity-based decision-making framework for FFHSs and developed an FFHS-TOPSIS method for solving multidimensional uncertain decision problems. The proposed approach satisfies the required axiomatic properties and effectively preserves the complex interrelationships among hypersoft attributes. The bridge design case study demonstrated that the proposed FFHS-Cosine method provides superior closeness/score values, stronger discrimination, and more reliable ranking than the compared methods. The results in Tables 6–10 confirm its better performance, perfect or near-perfect Spearman correlation, and stability under variations. Hence, the proposed framework is computationally efficient, mathematically sound, and practically useful for sustainable infrastructure decision-making. Future work may extend this model to neutrosophic hypersoft environments, group decision-making, and real-world large-scale applications.

CRediT authorship contribution statement

Zaffar Ahmed Shaikh: Writing – original draft, Validation, Supervision, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Muhammad Naveed Jafar:** Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Ali Elrashidi:** Writing – original draft, Validation, Supervision, Resources, Methodology, Investigation, Formal analysis, Conceptualization. **Muhammad Ahmad:** Writing – review & editing, Visualization, Validation, Resources, Methodology, Investigation, Formal analysis. **Gabriel Xiao-Guang Yue:** Writing – review & editing, Visualization, Validation, Methodology, Investigation, Formal analysis.

Funding statement

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. Accordingly, no funding body was involved in the study design, data analysis, interpretation of results, manuscript preparation, or publication decision.

Conflict of interests

The authors have no conflicts of interest to declare. All co-authors have seen and agree with the contents of the manuscript, and there is no financial interest to report.

Data availability

Data will be made available on request.

References

- [1] Zordan, T., Briseghella, B., Mazzarolo, E., Feb. 2010. Bridge structural optimization through step-by-step evolutionary process. *Struct. Eng. Int.* 20 (1), 72–78. <https://doi.org/10.2749/101686610791555586>.
- [2] Zhou, Z.W., Alcalá, J., Yepes, V., 2022. Regional sustainable development impact through sustainable bridge optimization. In: *Structures*. Elsevier, pp. 1061–1076 [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S235201242200409X>. (Accessed 20 April 2026).
- [3] Zaheer, Q., Yonggang, T., Qamar, F., 2022. Literature review of bridge structure's optimization and its development over time. *Int. J. Simul. Multidiscip. Des. Optim.* 13, 5.
- [4] Xin, H., Mosallam, A., Correia, J.A., Liu, Y., He, J., Sun, Y., 2020. Material-structure integrated design optimization of GFRP bridge deck on steel girder. In: *Structures*. Elsevier, pp. 1222–1230 [Online]. Available: <https://www.sciencedirect.com/science/article/pii/S2352012420303210>. (Accessed 20 April 2026).
- [5] Wang, Y., Ma, C., Wang, C., Zhang, H., Fan, S., Mar. 2023. Characteristics analysis and optimization design of bridge crane based on improved particle swarm optimization algorithm. *J. Low Freq. Noise Vib. Act. Control* 42 (1), 253–271. <https://doi.org/10.1177/14613484221118994>.
- [6] Trentini, E.V.W., Parsekian, G.A., Bittencourt, T.N., 2025. Multi-objective optimization for bridge and viaduct design: case study. *Rev. IBRACON Estrut. Mater.* 18, e18104.
- [7] Tlustochowicz, G., 2005. Optimized design of integral abutments for a three span composite bridge [Online]. Available: <https://www.diva-portal.org/smash/record.jsf?pid=diva2:1025325>. (Accessed 20 April 2026).
- [8] Templeman, A.B., Oct. 1983. Optimization methods in structural design practice. *J. Struct. Eng.* 109 (10), 2420–2433. [https://doi.org/10.1061/\(ASCE\)0733-9445\(1983\)109:10\(2420\)](https://doi.org/10.1061/(ASCE)0733-9445(1983)109:10(2420)).
- [9] Sengupta, A., Pereira, T., Liserre, M., 2024. Design optimization of dual active bridge converter for supercapacitor application. *IEEE Trans. Power Electron.* 39 (9), 11544–11557.
- [10] Penadés-Pla, V., García-Segura, T., Yepes, V., 2020. Robust design optimization for low-cost concrete box-girder bridge. *Mathematics* 8 (3), 398.
- [11] Papapetrou, V.S., Tamijani, A.Y., Brown, J., Kim, D., Feb. 2019. Design optimization of hybrid FRP/RC bridge. *Appl. Compos. Mater.* 26 (1), 249–270. <https://doi.org/10.1007/s10443-018-9691-3>.
- [12] Miguel, L.F., Lopez, R.H., Torii, A.J., Miguel, L.F., Beck, A.T., 2016. Robust design optimization of TMDs in vehicle-bridge coupled vibration problems. *Eng. Struct.* 126, 703–711.
- [13] Lounis, Z., Cohn, M.Z., Jul. 1995. An engineering approach to multicriteria optimization of bridge structures. *Comput. Aided Civ. Infrastruct. Eng.* 10 (4), 233–238. <https://doi.org/10.1111/j.1467-8667.1995.tb00285.x>.
- [14] Islam, N., Rana, S., Ahsan, R., Ghani, S.N., Feb. 2014. An optimized design of network arch bridge using global optimization algorithm. *Adv. Struct. Eng.* 17 (2), 197–210. <https://doi.org/10.1260/1369-4332.17.2.197>.
- [15] Gui, X., Zheng, X.Y., Song, J.W., Peng, X., 2011. Automation bridge design and structural optimization. *Appl. Mech. Mater.* 63, 457–460.
- [16] Feng, Y., Wang, C., Briseghella, B., Fenu, L., Zordan, T., Jul. 2021. Structural optimization of a steel arch bridge with genetic algorithm. *Struct. Eng. Int.* 31 (3), 347–356. <https://doi.org/10.1080/10168664.2020.1773373>.
- [17] Clingenpeel, B.F., Barth, K.E., 2003. Design optimization study of a three-span continuous bridge using HPS70W. *Eng. J.* 40 (3), 149–158.
- [18] Briseghella, B., Fenu, L., Feng, Y., Lan, C., Mazzarolo, E., Zordan, T., Mar. 2016. Optimization indexes to identify the optimal design solution of shell-supported bridges. *J. Bridge Eng.* 21 (3), 04015067. [https://doi.org/10.1061/\(ASCE\)BE.1943-5592.0000838](https://doi.org/10.1061/(ASCE)BE.1943-5592.0000838).
- [19] Cao, H., Qian, X., Chen, Z., Zhu, H., 2017. Layout and size optimization of suspension bridges based on coupled modelling approach and enhanced particle swarm optimization. *Eng. Struct.* 146, 170–183.
- [20] Cid Montoya, M., Feb. 2024. Sequential aero-structural optimization method for efficient bridge design. *Comput. Aided Civ. Infrastruct. Eng.* 39 (3), 319–344. <https://doi.org/10.1111/mice.13005>.
- [21] Olanrewaju, O., Oladele, I.O., Adelani, S.O., Sep. 2025. A comprehensive review on plant and animal fiber reinforced composites: experimental and theoretical approaches to interfacial strength optimization and potential applications. *Hybrid Adv.* 10, 100474. <https://doi.org/10.1016/j.hybadv.2025.100474>.
- [22] Sikder, S., Hasan, Md K., Mamur, H., Bhuiyan, M.R.A., Sep. 2025. Optimizing layer configuration and material selection to enhance CIGS solar cell performance through computational simulation. *Hybrid Adv.* 10, 100460. <https://doi.org/10.1016/j.hybadv.2025.100460>.
- [23] Husna, A., Ashrafi, S., Tomal, A.A., Tuli, N.T., Bin Rashid, A., Dec. 2024. Recent advancements in stereolithography (SLA) and their optimization of process parameters for sustainable manufacturing. *Hybrid Adv.* 7, 100307. <https://doi.org/10.1016/j.hybadv.2024.100307>.
- [24] Sinkhonde, D., et al., Mar. 2026. A novel machine learning approach for predicting the compressive strength of high-strength concrete containing gum Arabic and corn cob ash. *Hybrid Adv.* 12, 100579. <https://doi.org/10.1016/j.hybadv.2025.100579>.

- [25] Magombe, M., Yiga, V.A., Turyasingura, M., Okure, M., Lubwama, M., Mar. 2026. Optimization of calcium silicate based high density structural flame-retardant composites developed using waste marble powder and sugarcane bagasse ash for flame retardancy applications. *Hybrid Adv.* 12, 100630. <https://doi.org/10.1016/j.hybadv.2026.100630>.
- [26] Zadeh, L.A., Jun. 1965. Fuzzy sets. *Inf. Control* 8 (3), 338–353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X).
- [27] Atanassov, K.T., Aug. 1986. Intuitionistic fuzzy sets. *Fuzzy Set Syst.* 20 (1), 87–96. [https://doi.org/10.1016/S0165-0114\(86\)80034-3](https://doi.org/10.1016/S0165-0114(86)80034-3).
- [28] Yager, R.R., 2013. Pythagorean fuzzy subsets. In: 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS). IEEE, pp. 57–61 [Online]. Available: <https://ieeexplore.ieee.org/abstract/document/6608375/>. (Accessed 16 October 2024).
- [29] Senapati, T., Yager, R.R., Feb. 2020. Fermatean fuzzy sets. *J. Ambient Intell. Hum. Comput.* 11 (2), 663–674. <https://doi.org/10.1007/s12652-019-01377-0>.
- [30] Molodtsov, D., 1999. Soft set theory—first results. *Comput. Math. Appl.* 37 (4–5), 19–31.
- [31] Roy, A.R., Maji, P.K., Jun. 2007. A fuzzy soft set theoretic approach to decision making problems. *J. Comput. Appl. Math.* 203 (2), 412–418. <https://doi.org/10.1016/j.cam.2006.04.008>.
- [32] Smarandache, F., 2018. Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. *Neutrosophic sets syst.* 22 (1), 168–170.
- [33] Jafar, M., Saeed, M., Sep. 2021. Aggregation Operators of Fuzzy Hypersoft Sets, vol. 11, pp. 1–17.
- [34] Jafar, M.N., Saeed, M., Haseeb, M., Habib, A., 2021. Matrix theory for intuitionistic fuzzy hypersoft sets and its application in multi-attributive decision-making problems. *Theory and Application of Hypersoft Set* 65. (Accessed 14 May 2025).
- [35] Jafar, M.N., Saeed, M., 2022. Matrix theory for neutrosophic hypersoft set and applications in multiattributive multicriteria decision-making problems. *J. Math.* 2022, 1–15.
- [36] Saeed, M., Wahab, A., Ali, M., Ali, J., Bonyah, E., Sep. 2023. An innovative approach to passport quality assessment based on the possibility q-rung ortho-pair fuzzy hypersoft set. *Heliyon* 9 (9), e12345. <https://doi.org/10.1016/j.heliyon.2023.e19379>.
- [37] Khan, S., Gulistan, M., Kausar, N., Kadry, S., Kim, J., Nov. 2023. A novel method for determining tourism carrying capacity in a decision-making context using q-Rung orthopair fuzzy hypersoft environment. *CMES - Computer Modeling in Engineering and Sciences* 138 (2), 1951–1979. <https://doi.org/10.32604/cmcs.2023.030896>.
- [38] Khan, S., Gulistan, M., Kausar, N., Kousar, S., Pamucar, D., Addis, G.M., 2022. Analysis of cryptocurrency market by using q-Rung orthopair fuzzy hypersoft set algorithm based on aggregation operators. *Complexity* 2022 (1), 7257449. <https://doi.org/10.1155/2022/7257449>.
- [39] Khan, S., Gulistan, M., Wahab, H.A., 2021. Development of the structure of q-Rung orthopair fuzzy hypersoft set with basic operations. *Punjab Univ. J. Math.* 53 (12), 881–892.
- [40] Khan, S., Gulistan, M., Kausar, N., Pamucar, D., Ozbilge, E., El-Kanj, N., Feb. 2023. q-Rung orthopair fuzzy hypersoft ordered aggregation operators and their application towards green supplier. *Front. Environ. Sci.* 10. <https://doi.org/10.3389/fenvs.2022.1048019>.
- [41] Rahman, A.U., Saeed, M., Smarandache, F., Ahmad, M.R., 2020. Development of hybrids of hypersoft set with complex fuzzy set, complex intuitionistic fuzzy set and complex neutrosophic set. *Neutrosophic sets syst.* 38, 1. https://digitalrepository.unm.edu/nss_journal/vol38/iss1/22.
- [42] Nafei, A., Huang, C.Y., Javadpour, A., Garg, H., Azizi, S.P., Chen, S.C., Apr. 2024. Neutrosophic fuzzy decision-making using TOPSIS and autocratic methodology for machine selection in an industrial factory. *Int. J. Fuzzy Syst.* 26 (3), 860–886. <https://doi.org/10.1007/s40815-023-01640-9>.
- [43] Samad, A., et al., Jan. 2021. Selection of an effective hand sanitizer to reduce COVID-19 effects and extension of TOPSIS technique based on correlation coefficient under neutrosophic hypersoft set. *Complexity* 2021 (1), 5531830. <https://doi.org/10.1155/2021/5531830>.
- [44] Zulqarnain, R.M., Ma, W.-X., Siddique, I., Ahmad, H., Askar, S., 2024. A fair bed allocation during COVID-19 pandemic using TOPSIS technique based on correlation coefficient for interval-valued pythagorean fuzzy hypersoft set. *Sci. Rep.* 14 (1), 7678.
- [45] Javadpour, A., Nafei, A., Ja'fari, F., Pinto, P., Zhang, W., Sangaiah, A.K., Apr. 2023. An intelligent energy-efficient approach for managing IoT tasks in cloud platforms. *J. Ambient Intell. Hum. Comput.* 14 (4), 3963–3979. <https://doi.org/10.1007/s12652-022-04464-x>.
- [46] Zulqarnain, Rana Muhammad, Xin, Xiao Long, Ali, Bagh, Broumi, Said, Abdal, Sohaib, Ahamad, Muhammad Irfan, 2021. Decision-Making Approach Based on Correlation Coefficient with its Properties Under Interval-Valued Neutrosophic Hypersoft Set Environment [Online]. Available: <http://archive.org/details/13-decision-making-approach-based-on-correlation-2>. (Accessed 25 May 2026).
- [47] Zulqarnain, R.M., Xin, X.L., Saqlain, M., Saeed, M., Smarandache, F., Ahamad, M.I., 2021. Some fundamental operations on interval valued neutrosophic hypersoft set with their properties. *Neutrosophic sets and systems* 40 (1), 8.
- [48] Zulqarnain, R.M., Siddique, I., Mahboob, A., Ahmad, H., Askar, S., Gurmani, S.H., 2023. Optimizing construction company selection using einstein weighted aggregation operators for q-rung orthopair fuzzy hypersoft set. *Sci. Rep.* 13 (1), 6511.
- [49] Zulqarnain, R.M., et al., 2022. Einstein ordered weighted aggregation operators for pythagorean fuzzy hypersoft set with its application to solve MCDM problem. *IEEE Access* 10, 95294–95320.
- [50] Siddique, I., Zulqarnain, R.M., Ali, R., Jarad, F., Iampan, A., Jan. 2021. Multicriteria decision-making approach for aggregation operators of pythagorean fuzzy hypersoft sets. *Comput. Intell. Neurosci.* 2021 (1), 2036506. <https://doi.org/10.1155/2021/2036506>.