Application ID:

NAME:

2020 INHA-BEU DDP Test Answer Sheet for Math

1. [5 points]

$\sqrt{x+2} + \sqrt{8-x} = 4$

$(\sqrt{8-x})^2 = (4-\sqrt{x+2})^2$

$4\sqrt{x+2} = 5+x,\ (4\sqrt{x+2})^2 = (5+x)^2$

$\therefore x^2-6x-7=0.$

Therefore $x=-1,\ 7.$

The sum of solutions $(-1)+7=6.$

Answer: 6

2. [5 points]

$\left(a+\frac{3}{a}\right)\left(3a+\frac{1}{a}\right)=3a^2+\frac{3}{a^2}+10$

$=3(a^2+\frac{1}{a^2})+10 \geq 6\sqrt{a^2\cdot\frac{1}{a^2}}+10 = 16$.

Therefore the minimum value is 16.

Answer: 16

3. [10 points]

$\log_2(x^2+5x+12) - \log_2(x+5) =$

$\log_2\frac{x^2+5x+12}{x+5} = \log_2 2$. $\therefore \frac{x^2+5x+12}{x+5} = 2$.

$\therefore x^2+5x+12 = 2(x+5)$. $\therefore x^2+3x+2 = 0$

$\therefore (x+2)(x+1) = 0$.

Therefore, $x = -2,\ -1$.

The sum of solutions is -3 .

Answer: -3

4. [10 points]

$$ctg\left(\frac{\pi}{12}\right) = ctg\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}$$

$$=\frac{\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}}{\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}}=\frac{\frac{1}{2}\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}-\frac{1}{2}\frac{\sqrt{2}}{2}}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{\sqrt{6}-\sqrt{2}}=\frac{8+4\sqrt{3}}{4}=2+\sqrt{3}.$$

Answer: $2+\sqrt{3}$

5. [15 points]

$t = x-\frac{\pi}{4} \Rightarrow x = t+\frac{\pi}{4}$

$$\cos 2x = \cos\left(2t+\frac{\pi}{2}\right)$$

$$= \cos 2t\cos\frac{\pi}{2} - \sin 2t\sin\frac{\pi}{2}$$

$$= -\sin 2t = -2\sin t\cos t$$

$$\lim_{x\to\frac{\pi}{4}}\frac{x-\frac{\pi}{4}}{\cos 2x} = \lim_{t\to 0}\frac{t}{-2\sin t\cos t}$$

$$=\lim_{t\to 0}-\frac{1}{2}\frac{t}{\sin t}\frac{1}{\cos t} = -\frac{1}{2}\cdot 1\cdot 1 = -\frac{1}{2}.$$

Answer: $-\frac{1}{2}$

6. [15 points]

$$f'(x) = 2020(2x+1)^{2019}\cdot 2\cdot(5x^2-7x+2)$$
$$+(2x+1)^{2020}\cdot(10x-7).$$

$$f'(0) = 2020\cdot 2\cdot 2+1\cdot(-7) = 8080-7$$
$$=8073.$$

Answer: 8073